

```

┆  $\forall [P, Q: \mathbb{P}]. ((P \Rightarrow Q) \wedge ((P \vee (\neg P)) \vee Q \vee (\neg Q))) \Rightarrow ((\neg P) \vee Q)$ 
|
BY RepeatFor 2 ((UD THENA Auto))
|
[1]. P:  $\mathbb{P}$ 
[2]. Q:  $\mathbb{P}$ 
┆  $((P \Rightarrow Q) \wedge ((P \vee (\neg P)) \vee Q \vee (\neg Q))) \Rightarrow ((\neg P) \vee Q)$ 
|
BY (D 0 THENA Auto)
|
3.  $(P \Rightarrow Q) \wedge ((P \vee (\neg P)) \vee Q \vee (\neg Q))$ 
┆  $(\neg P) \vee Q$ 
|
BY D 3
|
3.  $P \Rightarrow Q$ 
4.  $(P \vee (\neg P)) \vee Q \vee (\neg Q)$ 
┆  $(\neg P) \vee Q$ 
|
BY D 4
| \
| 4.  $P \vee (\neg P)$ 
| ┆  $(\neg P) \vee Q$ 
| |
1 BY D 4
| | \
| | 4. P
| | ┆  $(\neg P) \vee Q$ 
| | |
1 2 BY (OrRight THENA Auto)
| | |
| | ┆ Q
| | |
1 2 BY D 3
| | | \
| | | 3. P
| | | ┆ P
| | | |
1 2 3 BY NthHyp 3
| | | \
| | | 3. P
| | | 4. Q
| | | ┆ Q
| | | |
1 2 BY NthHyp 4
| | \
| | 4.  $\neg P$ 
| | ┆  $(\neg P) \vee Q$ 
| | |
1 BY (OrLeft THENA Auto)
| |
| | ┆  $\neg P$ 
| |

```

```

1  BY NthHyp 4
  \
  4. Q  $\vee$  ( $\neg$ Q)
   $\vdash$  ( $\neg$ P)  $\vee$  Q
  |
  BY D 4
  | \
  | 4. Q
  |  $\vdash$  ( $\neg$ P)  $\vee$  Q
  | |
  1 BY (OrRight THENA Auto)
  | |
  |  $\vdash$  Q
  | |
  1 BY NthHyp 4
  | \
  | 4.  $\neg$ Q
  |  $\vdash$  ( $\neg$ P)  $\vee$  Q
  |
  BY (OrLeft THENA Auto)
  |
   $\vdash$   $\neg$ P
  |
  BY (D 0 THENA Auto)
  |
  5. P
   $\vdash$  False
  |
  BY D 3
  | \
  | 3.  $\neg$ Q
  | 4. P
  |  $\vdash$  P
  | |
  1 BY NthHyp 4
  | \
  | 3.  $\neg$ Q
  | 4. P
  | 5. Q
  |  $\vdash$  False
  |
  BY (Unfold 'not' 3 THEN D 3)
  | \
  | 3. P
  | 4. Q
  |  $\vdash$  Q
  | |
  1 BY NthHyp 4
  | \
  | 3. P
  | 4. Q
  | 5. False
  |  $\vdash$  False
  |
  
```

BY NthHyp 5

```
Extract:  $\lambda f.$ let pq,g = f in
  case g of inl(gp) => case gp of inl(p1) => inr (pq p1) | inr(np) => inl np
    | inr(gq) => case gq of inl(q) => inr q | inr(nq) => inl ( $\lambda p2.$ (nq (pq p2)))

where f : (P  $\Rightarrow$  Q)  $\wedge$  ((P  $\vee$  ( $\neg$ P))  $\vee$  Q  $\vee$  ( $\neg$ Q))
  pq : P  $\Rightarrow$  Q
  g : (P  $\vee$  ( $\neg$ P))  $\vee$  Q  $\vee$  ( $\neg$ Q)
  gp : (P  $\vee$  ( $\neg$ P))
  p1 : P
  np :  $\neg$ P  $\equiv$  (P  $\Rightarrow$  False)
  gq : Q  $\vee$  ( $\neg$ Q)
  q : Q
  nq :  $\neg$ Q  $\equiv$  (Q  $\Rightarrow$  False)
  p2 : P
```