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 $\vdash \forall p, q : \mathbb{N}^+. \forall L : \mathbb{Z} \text{ List}. (\text{seg\_sum}(p; q; L) = \text{seg\_sum}(p - 1; q - 1; \text{tl}(L)))$ 
|
BY RepeatFor 3 ((D 0 THENA Auto))
|
1. p:  $\mathbb{N}^+$ 
2. q:  $\mathbb{N}^+$ 
3. L:  $\mathbb{Z}$  List
 $\vdash \text{seg\_sum}(p; q; L) = \text{seg\_sum}(p - 1; q - 1; \text{tl}(L))$ 
|
BY Assert  $\lceil \exists x : \mathbb{Z}. (x = \text{seg\_sum}(p - 1; q - 1; \text{tl}(L))) \rceil.$ 
| \
|  $\vdash \exists x : \mathbb{Z}. (x = \text{seg\_sum}(p - 1; q - 1; \text{tl}(L)))$ 
| |
1 BY (InstConcl  $\lceil \text{seg\_sum}(p - 1; q - 1; \text{tl}(L)) \rceil$ ). THEN Auto
\ \
4.  $\exists x : \mathbb{Z}. (x = \text{seg\_sum}(p - 1; q - 1; \text{tl}(L)))$ 
 $\vdash \text{seg\_sum}(p; q; L) = \text{seg\_sum}(p - 1; q - 1; \text{tl}(L))$ 
|
BY D 4
|
4. x:  $\mathbb{Z}$ 
5.  $x = \text{seg\_sum}(p - 1; q - 1; \text{tl}(L))$ 
 $\vdash \text{seg\_sum}(p; q; L) = \text{seg\_sum}(p - 1; q - 1; \text{tl}(L))$ 
|
BY (RevHypSubst 5 0 THENA Auto)
|
 $\vdash \text{seg\_sum}(p; q; L) = x$ 
|
BY RepUR ``seg_sum segment'' 0
|
 $\vdash \text{l\_sum}(\text{firstn}((q + 1) - p; \text{nth\_tl}(p; L))) = x$ 
|
BY RecUnfold `nth_tl` 0
|
 $\vdash \text{l\_sum}(\text{firstn}((q + 1) - p; \text{if } p \leq z 0 \text{ then } L \text{ else } \text{nth\_tl}(p - 1; \text{tl}(L)) \text{ fi })) = x$ 
|
BY AutoBoolCase  $\lceil p \leq z 0 \rceil.$ 
|
2.  $\neg(p \leq 0)$ 
3. q:  $\mathbb{N}^+$ 
4. L:  $\mathbb{Z}$  List
5. x:  $\mathbb{Z}$ 
6.  $x = \text{seg\_sum}(p - 1; q - 1; \text{tl}(L))$ 
 $\vdash \text{l\_sum}(\text{firstn}((q + 1) - p; \text{nth\_tl}(p - 1; \text{tl}(L)))) = x$ 
|
BY Assert  $\lceil ((q + 1) - p) = (((q - 1) + 1) - p - 1) \rceil.$ 
| \
|  $\vdash ((q + 1) - p) = (((q - 1) + 1) - p - 1)$ 
| |
1 BY Auto
\ \
7.  $((q + 1) - p) = (((q - 1) + 1) - p - 1)$ 
 $\vdash \text{l\_sum}(\text{firstn}((q + 1) - p; \text{nth\_tl}(p - 1; \text{tl}(L)))) = x$ 

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$$\forall p, q : \mathbb{N}^+. \forall L : \mathbb{Z} \text{ List}. (\text{seg\_sum}(p; q; L) = \text{seg\_sum}(p - 1; q - 1; \text{tl}(L)))$$

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|  
| BY (HypSubst 7 0 THENA Auto)  
|  
| ⊢ l_sum(firstn((q - 1) + 1) - p - 1; nth_tl(p - 1; tl(L))) = x  
|  
| BY (Fold ‘segment’ 0 THEN Fold ‘seg_sum’ 0)  
|  
| ⊢ seg_sum(p - 1; q - 1; tl(L)) = x  
|  
| BY Auto
```