

```

⊢ ∀[T:Type]. ∀[A:T → ℙ]. {∃x:T. (A x) ⇔ ¬(∀x:T. (¬(A x)))}
|
BY (D 0 THENA Auto)
| \
| 1. T: Type
| ⊢ ∀[A:T → ℙ]. {∃x:T. (A x) ⇔ ¬(∀x:T. (¬(A x)))}
| |
1 BY (D 0 THENA Auto)
| | \
| | 2. A: T → ℙ
| | ⊢ {∃x:T. (A x) ⇔ ¬(∀x:T. (¬(A x)))}
| | |
1 2 BY (D 0 THENA Auto)
| | |
| | ⊢ {(∃x:T. (A x)) ⇒ ¬(∀x:T. (¬(A x)))} ∧ {(∃x:T. (A x)) ⇔ ¬(∀x:T. (¬(A x)))}
| | |
1 2 BY (RepeatFor 2 (D 0) THENA Auto)
| | | \
| | | ⊢ (∃x:T. (A x)) ⇒ {¬(∀x:T. (¬(A x)))}
| | | |
1 2 3 BY (D 0 THENA Auto)
| | | |
| | | 3. ∃x:T. (A x)
| | | ⊢ {¬(∀x:T. (¬(A x)))}
| | | |
1 2 3 BY (ElimClassical THENA Auto)
| | | |
| | | ⊢ ¬(∀x:T. (¬(A x)))
| | | |
1 2 3 BY (D 0 THENA Auto)
| | | |
| | | 4. ∀x:T. (¬(A x))
| | | ⊢ False
| | | |
1 2 3 BY D 3
| | | |
| | | 3. x: T
| | | 4. A x
| | | 5. ∀x:T. (¬(A x))
| | | ⊢ False
| | | |
1 2 3 BY (InstHyp ["x"] 5. THENA Auto)
| | | |
| | | 6. ¬(A x)
| | | ⊢ False
| | | |
1 2 3 BY D 6
| | | |
| | | ⊢ A x
| | | |
1 2 3 BY Hypothesis
| | \
| | ⊢ (¬(∀x:T. (¬(A x)))) ⇒ {∃x:T. (A x)}
| | |
1 2 BY (D 0 THENA Auto)
| | |

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| | 3.  $\neg(\forall x:T. (\neg(A x)))$ 
| |  $\vdash \{\exists x:T. (A x)\}$ 
| | |
1 2 BY (ClassicalContradiction THENA Auto)
| | |
| | 4.  $\neg(\exists x:T. (A x))$ 
| |  $\vdash \{\exists x:T. (A x)\}$ 
| | |
1 2 BY D 3
| | |
| | 3.  $\neg(\exists x:T. (A x))$ 
| |  $\vdash \forall x:T. (\neg(A x))$ 
| | |
1 2 BY (D 0 THENA Auto)
| | |
| | 4. x: T
| |  $\vdash \neg(A x)$ 
| | |
1 2 BY (D 0 THENA Auto)
| | |
| | 5. A x
| |  $\vdash \text{False}$ 
| | |
1 2 BY D 3
| | |
| | 3. x: T
| | 4. A x
| |  $\vdash \exists x:T. (A x)$ 
| | |
1 2 BY (InstConcl  $[\![x]\!]$ . THENA Auto)
| | |
| |  $\vdash A x$ 
| | |
1 2 BY Hypothesis
| \
| 2. A: T  $\rightarrow \mathbb{P}$ 
| 3.  $\{x:\text{Unit} \mid \exists x:T. (A x) \iff \neg(\forall x:T. (\neg(A x)))\}$ 
|  $\vdash Ax \in \{x:\text{Unit} \mid \exists x:T. (A x) \iff \neg(\forall x:T. (\neg(A x)))\}$ 
| |
1 BY Auto
\
1. T: Type
2. A: T  $\rightarrow \mathbb{P}$ 
3.  $\{x:\text{Unit} \mid \exists x:T. (A x) \iff \neg(\forall x:T. (\neg(A x)))\}$ 
 $\vdash Ax \in \{x:\text{Unit} \mid \exists x:T. (A x) \iff \neg(\forall x:T. (\neg(A x)))\}$ 
|
BY Auto

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