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⊢ ∀[T:Type]. ∀[A:ℙ]. ∀[B:T → ℙ]. {A ∨ (∀x:T. (B x))} ⇔ ∀x:T. (A ∨ (B x))}
|
BY (D 0 THENA Auto)
| \
| 1. T: Type
| ⊢ ∀[A:ℙ]. ∀[B:T → ℙ]. {A ∨ (∀x:T. (B x))} ⇔ ∀x:T. (A ∨ (B x))}
| |
1 BY (D 0 THENA Auto)
| | \
| | 2. A: ℙ
| | ⊢ ∀[B:T → ℙ]. {A ∨ (∀x:T. (B x))} ⇔ ∀x:T. (A ∨ (B x))}
| | |
1 2 BY (D 0 THENA Auto)
| | | \
| | | 3. B: T → ℙ
| | | ⊢ {A ∨ (∀x:T. (B x))} ⇔ ∀x:T. (A ∨ (B x))}
| | | |
1 2 3 BY RepeatFor 4 ((D 0 THENA Auto))
| | | | \
| | | | 4. A ∨ (∀x:T. (B x))
| | | | ⊢ {∀x:T. (A ∨ (B x))}
| | | | |
1 2 3 4 BY (ElimClassical THENA Auto)
| | | | |
| | | | ⊢ ∀x:T. (A ∨ (B x))
| | | | |
1 2 3 4 BY (D 0 THENA Auto)
| | | | |
| | | | 5. x: T
| | | | ⊢ A ∨ (B x)
| | | | |
1 2 3 4 BY D 4
| | | | | \
| | | | | 4. A
| | | | | ⊢ A ∨ (B x)
| | | | | |
1 2 3 4 5 BY (OrLeft THENA Auto)
| | | | | |
| | | | | | ⊢ A
| | | | | | |
1 2 3 4 5 BY Hypothesis
| | | | | \
| | | | | 4. ∀x:T. (B x)
| | | | | ⊢ A ∨ (B x)
| | | | | |
1 2 3 4 BY (OrRight THENA Auto)
| | | | | |
| | | | | | ⊢ B x
| | | | | | |
1 2 3 4 BY (InstHyp [x] 4. THENA Auto)
| | | | | |
| | | | | | 6. B x
| | | | | | ⊢ B x
| | | | | | |
1 2 3 4 BY Hypothesis
| | | | \

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| | | 4.  $\forall x:T. (A \vee (B x))$ 
| | |  $\vdash \{A \vee (\forall x:T. (B x))\}$ 
| | | |
1 2 3 BY (ClassicalContradiction THENA Auto)
| | | |
| | | 5.  $\neg(A \vee (\forall x:T. (B x)))$ 
| | |  $\vdash \{A \vee (\forall x:T. (B x))\}$ 
| | | |
1 2 3 BY (ElimClassical THENA Auto)
| | | |
| | |  $\vdash A \vee (\forall x:T. (B x))$ 
| | | |
1 2 3 BY (OrRight THENA Auto)
| | | |
| | |  $\vdash \forall x:T. (B x)$ 
| | | |
1 2 3 BY (D 0 THENA Auto)
| | | |
| | | 6.  $x: T$ 
| | |  $\vdash B x$ 
| | | |
1 2 3 BY (InstHyp [ $x$ ] 4. THENA Auto)
| | | |
| | | 7.  $A \vee (B x)$ 
| | |  $\vdash B x$ 
| | | |
1 2 3 BY D 7
| | | | \
| | | | 7.  $A$ 
| | | |  $\vdash B x$ 
| | | | |
1 2 3 4 BY D 5
| | | | |
| | | | 5.  $x: T$ 
| | | | 6.  $A$ 
| | | |  $\vdash A \vee (\forall x:T. (B x))$ 
| | | | |
1 2 3 4 BY (OrLeft THENA Auto)
| | | | |
| | | |  $\vdash A$ 
| | | | |
1 2 3 4 BY Hypothesis
| | | | \
| | | | 7.  $B x$ 
| | | |  $\vdash B x$ 
| | | | |
1 2 3 BY Hypothesis
| | | \
| | 3.  $B: T \rightarrow \mathbb{P}$ 
| | 4.  $\{x:\text{Unit} \mid A \vee (\forall x:T. (B x)) \iff \forall x:T. (A \vee (B x))\}$ 
| |  $\vdash Ax \in \{x:\text{Unit} \mid A \vee (\forall x:T. (B x)) \iff \forall x:T. (A \vee (B x))\}$ 
| | |
1 2 BY Auto
| | \
| 2.  $A: \mathbb{P}$ 
| 3.  $B: T \rightarrow \mathbb{P}$ 
| 4.  $\{x:\text{Unit} \mid A \vee (\forall x:T. (B x)) \iff \forall x:T. (A \vee (B x))\}$ 

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|   ⊢ Ax ∈ {x:Unit | A ∨ (∀x:T. (B x)) ⇔ ∀x:T. (A ∨ (B x))}
|   |
1  BY Auto
|   \
    1. T: Type
    2. A: ℙ
    3. B: T → ℙ
    4. {x:Unit | A ∨ (∀x:T. (B x)) ⇔ ∀x:T. (A ∨ (B x))}
    ⊢ Ax ∈ {x:Unit | A ∨ (∀x:T. (B x)) ⇔ ∀x:T. (A ∨ (B x))}
    |
    BY Auto

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