

*Nuprl-Light*

**Nuprl**



# *Outline*

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- **Intro to Nuprl**

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# *Example theory*

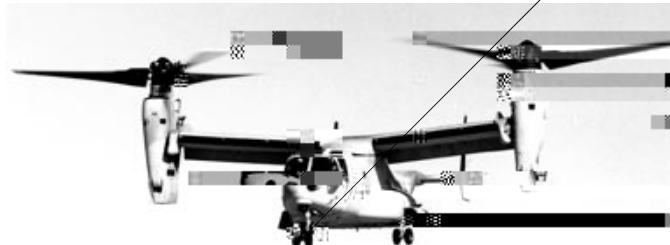
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Engine



Plane

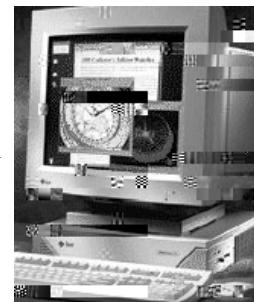
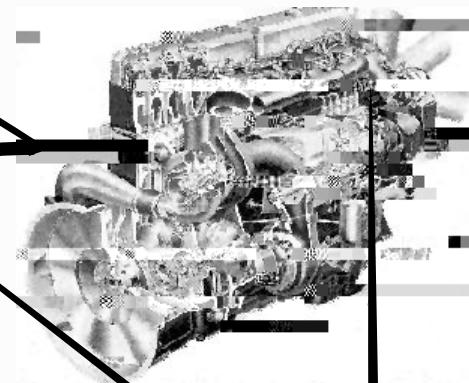


$\vdash \forall t: \mathbb{N}. \forall p: Plane. \Diamond$

library

overall architecture

Refiner  
(topic Engine)



# *Problems*

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- Want specialization

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# *Programming Languages*

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- Type systems (weak)

Module systems

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ute modules

4205 objects with:

- Subtyping
- Inheritance
- Reuse

# ***Formal Modules***

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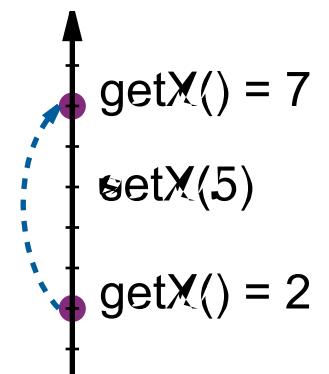
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First class (

# *Point Example*

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```
module Point =  
begin  
  include
```



# *Point Implementation*

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```
theory PointClass =
  begin
    (* I *)
    theorem can: Type = {x: Z}
    theorem getx: car → Z = λr.r · x
    theorem bumpx: car → Z = car = λr.λi.r · x := (r · x + i)
    theorem spec = λi.λp.Axiom
  end

  module ColorPointClass =
    begin
      pointClass(car = {x: Z; c: }): Point
      theorem Color: Type = {red, green, blue}
      theorem getc: car → Color = λr.r · c
      theorem setc: car → Color = car = λr.λcc.r · c :=
        spec = λp.λc. . .
    end
```

# *Algebra*

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**theory** *LeftMonoid*

**axiom**  $\oplus : \text{car} \rightarrow \text{car}$

**axiom**  $1 : \text{car}$

**axiom**  $\forall m : \text{car}. m \oplus 1 \equiv m$

...

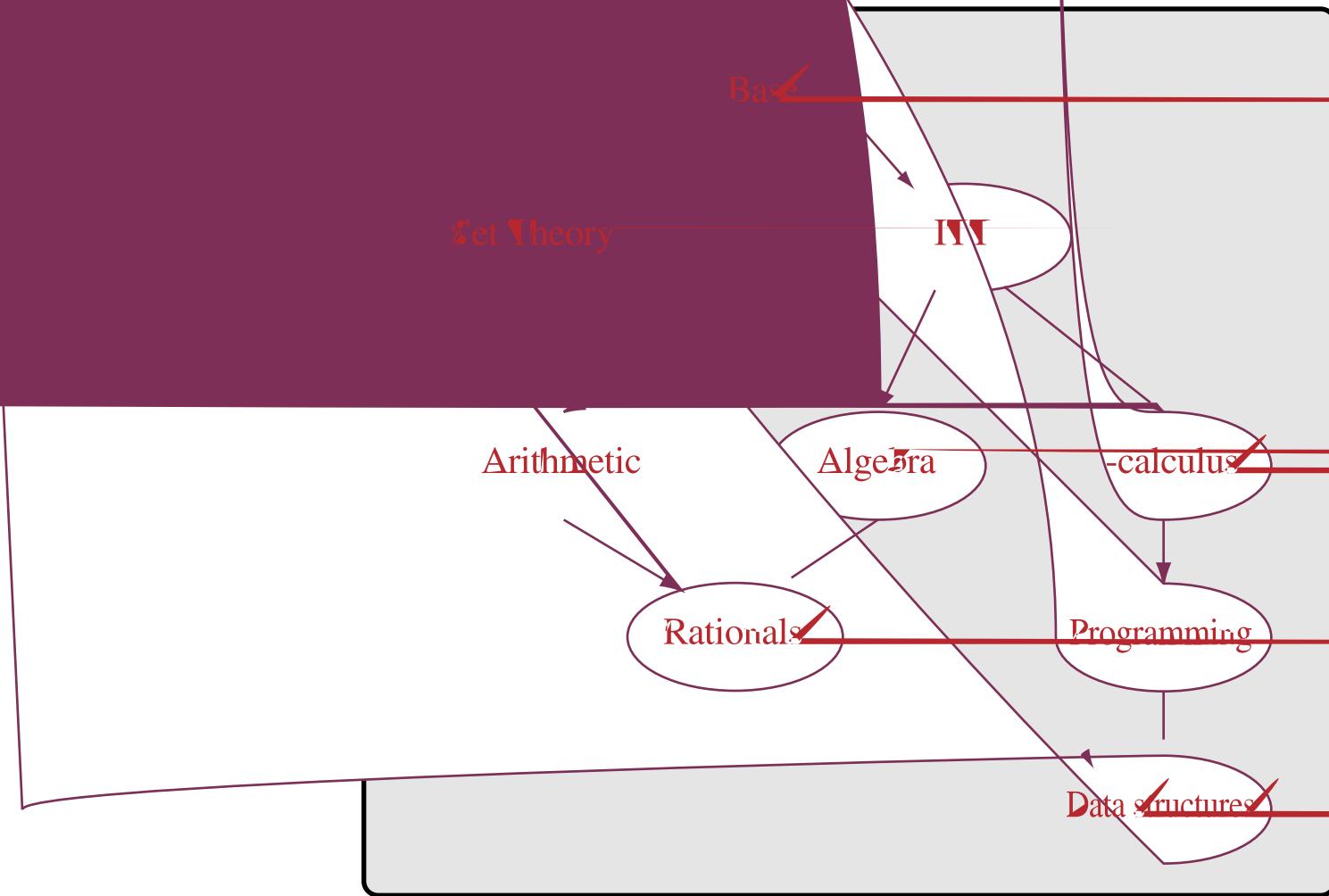


**theory** *Group* ~~*LeftMonoid*~~

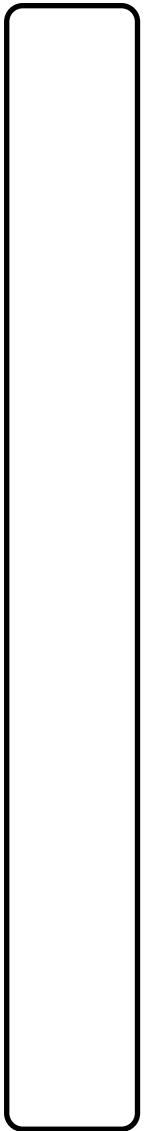
$\equiv$

# *Logical Framework*

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# *Features of the module system*

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- Multiple inheritance

Multiple display form collections

Automatic tactic generation from rules

Tactic joining for “D,” “Eq”

- Automatic generation of

```
(* Integers *)
include Itt_equal;;
include Itt_rfun;;
include Itt_logic;;
```

Dependencies

```
declare int;;
declare natural_number[$n n];;
...
val int term term;;
val zero term;;
declare "ass"{'a; 'b};;
...
```

Declarations

```
rewrite reduceAss "ass'{natural_number[$i n]; natural_number[$j n]} <->
  natural_number[$i + $j];;
...
rewrite inc reduceDown
  'x < 0 -->
  ((inc{'x; i, j. 'down[i; j]; 'base; k, l. 'up['k; 'l]})) <->
  ('down['x; inc{'x ass 1; i, j. 'down[i; j]; 'base; k, l. 'up['k; 'l]}]);;
```

Rewrites

```
(* 
 * Induction
 * H, n:Z, J[n] >> E[n] ext ind{i; m, z. down[n, m, it, z]; base[n]; m, z. up[n, m, it, z]}
 * by intElimination [m; v; z]
 *
 * H, n:Z, J[n], m:Z, v: m < 0, z: E[m + 1] >> E[m] ext down[n, m, v, z]
 * H, n:Z, J[n] >> E[0] ext base[n]
 * H, n:Z, J[n], m:Z, v: 0 < m, z: E[m - 1] >> E[m] ext up[n, m, v, z]
 *)

```

Rules

```
axiom intElimination 'H 'J 'n 'm 'v 'z
sequent { 'H; n. int; 'J['n]; m. int; v. 'm < 0; z. 'C['m ass 1] >> 'C['m] } -->
sequent { 'H; n. int; 'J['n] >> 'C[0] } -->
sequent { 'H; n. int; 'J['n]; m. int; v. 0 < 'm; z. 'C['m sub 1] >> 'C['m] } -->
sequent { 'H; n. int; 'J['n] >> 'C['n] };;
```

# *Primitives*

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```
axiom <name> <term> :-  
rewrite <name>  
declare <term>  
define <name> <term> <--> <term>
```

# *Implementation*

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- Refiner is implemented in Caml-Light
- “Refiner” is unit of truth
  - Rule set
  - Tactic/rewrite validation
  - Proof validation
  - Everythin

# *Editor*

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## *Future goals*

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Bring Caml and Nu