The Logic of Events, a framework to reason about distributed systems

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Summary

We have :

- A logical specification language (the logic of events) that formalizes the message sequence diagrams systems engineers use.
- A logical and compositional abstraction (event classes) from which we can synthesize code.
- A language (EventML) for defining event classes and their high-level properties.
- Automated tools that prove invariants and derive "inductive logical forms" that streamline the proofs of distributed algorithms.
  - In two days we now construct proofs of agreement and validity properties of a consensus algorithm.
  - Those proofs used to take a month to create.

Proofs as programs → Proofs as processes

- Programs are the evidence for Propositions.

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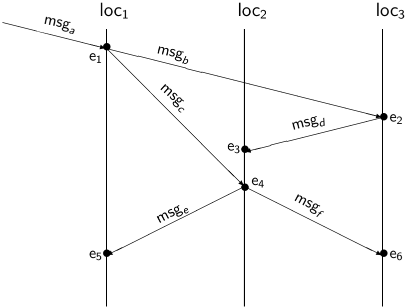
- Event ordering = $\langle E, loc(e), info(e), e_1 < e_2 \rangle$ + six axioms
- Event Logic = propositions in CTT about event orderings
- Evidence ?? could be IO-Automata, π-calculus, ...



Event class: the link to computation

An event class X of type $class(T)$ is both

- A relation $v \in X(e)$
  - X observes v at event e
  - X associates information v with event e
- A function $X : EO \rightarrow E \rightarrow \text{Bag}(T)$

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- $v \in Base(hdr,type)(e) \Leftrightarrow info(e) = \langle hdr, type, v \rangle$

Example: consensus safety properties

Agreement

If commands c and c' are chosen for the n^{th} command then $c = c'$.

```
∀e1,e2:E. ∀n:ℤ. ∀c,c':Cmd.
  <n, c> ∈ notify'base(e1)
  ⇒ <n, c'> ∈ notify'base(e2)
  ⇒ c = c'
```

Validity Any command decided on must have been proposed.

```
∀e:E. ∀n:ℤ. ∀c:Cmd.
 <n, c> ∈ notify'base(e)
  ⇒ (∃e':E. (e' < e) ∧
                 <n, c> ∈ propose'base(e'))
```

Event class combinators

(used here to structure 2/3 majority consensus algorithm)

```
main        = Replica @ locs

Replica     = NewVoters >>= \p.Voter p
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Voter (n,c)   = Round ((n,0),c)
                || (Notify n)
                || ((NewRounds n >>= Round)
                        until (Notify n))

Round (ni,c)  = SendVotes (ni,c)
                || Once(Quorum ni)
```

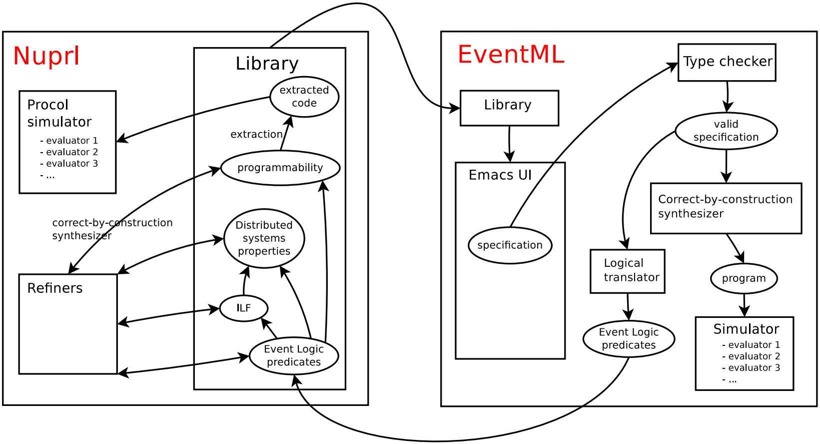
Event classes and combinators are expressible in EventML.

Computation and logic

Event classes have two facets:

- **computational**:
  - they can be implemented as processes (tail recursive)
  - program for each combinator derived from constituent programs
  - all constructions proved correct in Nuprl
  - result: a verified code synthesizer from event classes to processes
- **logical**:
  - they specify information flow (using the class relation)
  - relation for each combinator derived from constituent relations
  - derived relations proved correct in Nuprl
  - result: a verified translator from event classes to logical relations

Cooperation with a Logical Programming Environment (LPE)



EventML prelude

```
specification rsc4

(* ——— PARAMETERS ——— *)

(* consensus on commands of aribtrary type Cmd with equality decider *)
parameter Cmd, cmdeq : Type * Cmd Deq

parameter coeff   : Int
parameter flrs    : Int      (* max number of failures                *)
parameter locs    : Loc Bag  (* set of exactly (3 * flrs + 1) locations *)
parameter clients : Loc Bag  (* locations of the clients to be notified *)

(* ——— CONSTANTS ——— *)
import length poss-maj list-diff deq-member from-upto Memory-class
       int-list-member

(* ——— TYPE FUNCTIONS ——— *)

type Inning = Int
type CmdNum = Int
type CI     = CmdNum * Inning
type CC     = CmdNum * Cmd
type Vote   = (CI * Cmd) * Loc

(* ——— INTERFACE ——— *)

internal vote    : Vote
internal retry   : CI * Cmd
internal decided : CC
output   notify  : CC
input    propose : CC
```

EventML

```
(* -- inputs -- *)
let vote2prop loc (((n,i),c),loc') = {(n,c)} ;;
class Proposal = propose'base || (vote2prop o vote'base);;

(* -- output -- *)
let when_new_proposal loc (n,c) (max,missing) =
  if n > max or deq-member (op =) n missing then {(n,c)} else {} ;;

(* -- update -- *)
let update_replica (n,c) (max,missing) =
  if n > max
  then (n, missing ++ (from-upto (max + 1) n))
  else if deq-member (op =) n missing
  then (max, list-diff (op =) missing [n])
  else (max,missing) ;;

(* -- New votes state -- *)
class ReplicaState = Memory-class update_replica (init (0,nil)) Proposal ;;

(* -- New votes observer -- *)
class NewVoters = when_new_proposal o (Proposal, ReplicaState) ;;

(* ---------- Replica ---------- *)
class Replica = NewVoters >>= Voter;;

(* ---------- Main program ---------- *)
main Replica @ locs ;;
```

EventML assertions

```
(* -- state -- *)
class ReplicaState = Memory-class update_replica (init (0,nil)) Proposal ;;

(* -- invariants -- *)
invariant replica_inv on (max,missing) in ReplicaState
  == max >= 0
  /\ forall x : Int, int-list-member x missing => max > x /\ x > 0;;
```

Automated tactics prove many assertions automatically.

Inductive logical form (ILF)

automatically generated, automatically proved

```
∀[Cmd:ValueAllType]. ∀[clients:bag(Id)]. ∀[cmdeq:EqDecider(Cmd)]. ∀[coeff,flrs:ℤ]. ∀[locs:bag(Id)].
∀[es:EO']. ∀[e:E]. ∀[rcvr:Id]. ∀[num,rnd:ℤ]. ∀[c:Cmd]. ∀[sndr:Id].
  (<rcvr, rsc4_vote'msg(Cmd;<<<num, rnd>, c>, sndr>)> ∈ rsc4_Main(e)
  ⟺ loc(e) ∈ locs
      ∧ (rcvr ∈ locs ∧ (sndr = loc(e)))
      ∧ (∃e':{e':E| e' ≤loc e }
          ((∃max:ℤ
              ∃missing:ℤ List
               (<max, missing> ∈ rsc4_ReplicaState(Cmd)(e') ∧ ((max < num) ∨ (num ∈ missing))))
          ∧ (∃c':Cmd
              ((((e = e') ∧ (c = c') ∧ (rnd = 0))
                 ∨ ((∃e1:{e1:E| e1 ≤loc e }
                        (((∃maxr:ℤ. (maxr ∈ rsc4_NewRoundsState(Cmd) num(e1) ∧ (maxr < rnd)))
                        ∧ (<<num, rnd>, c> ∈ rsc4_retry'base(Cmd)(e1)
                          ∨ (∃sndr':Id. <<<num, rnd>, c>, sndr'> ∈ rsc4_vote'base(Cmd)(e1))))
                        ∧ (e = e1)))
                   ∧ (no rsc4_Notify(Cmd;clients) num between e' and e)))
              ∧ (<num, c'> ∈ rsc4_propose'base(Cmd)(e')
                ∨ (∃rnd':ℤ. ∃sndr':Id. <<<num, rnd'>, c'>, sndr'> ∈ rsc4_vote'base(Cmd)(e')))))))
```

Conclusion

The right abstractions, embedded in a language that can interface with automated theorem provers gives us the ability to synthesize code that provably satisfies high-level specifications.