

# Lecture 4

## Event Systems

This lecture is based on work done  
with Mark Bickford.

Marktoberdorf Summer School, 2003



# Formal Methods

One of the major research challenges faced by computer science is providing a technology for building **highly reliable software** systems that perform well and evolve economically to ever-higher reliability and better performance.

One approach is using **formal methods**, tools based on logic. They are not widely adopted. There are several approaches to improving them:

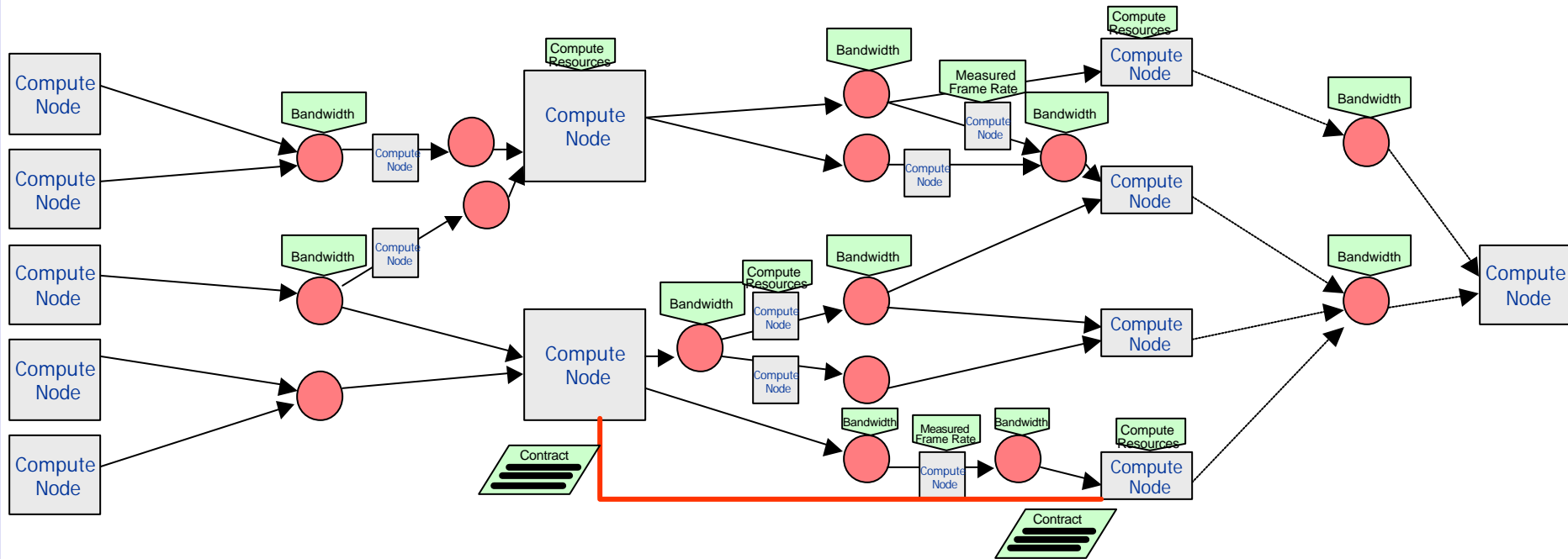
- More fully automated tools – type checkers, model checkers, decision procedures
- More effective interactive tools – provers

# Our Experience

We have worked on distributed system verification for a decade, with Ken Birman's group at Cornell.

We discovered that we have accumulated sufficient knowledge that we could participate in protocol design and development **at the speed of the designers** and programmers.

# Abstract Network Model – Media Net



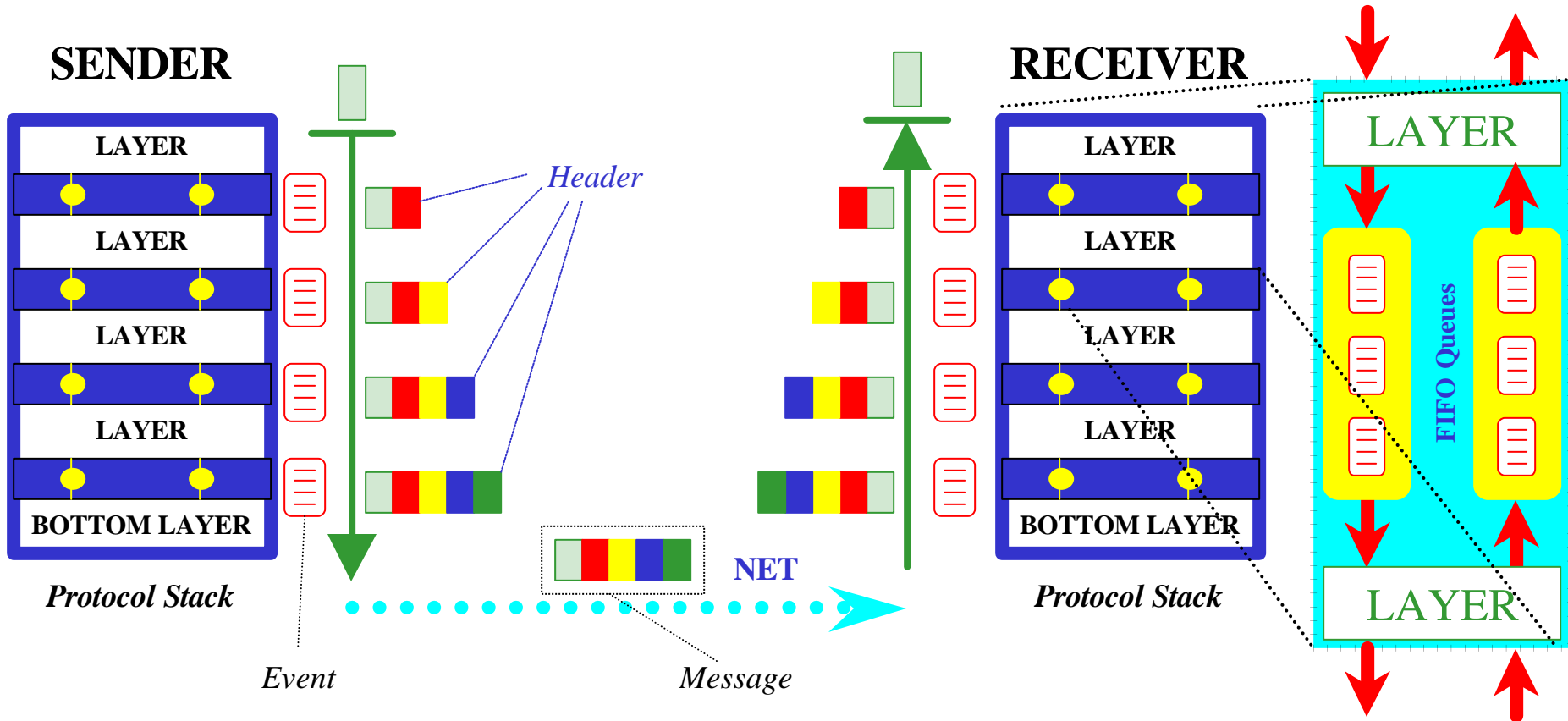
# Concurrency Models

There is an enormous literature providing models for concurrent computation and distributed systems, e.g. Petri nets, process calculi, operational semantics, domain theories, machine models (IO Automata, message automata), model theory, etc.

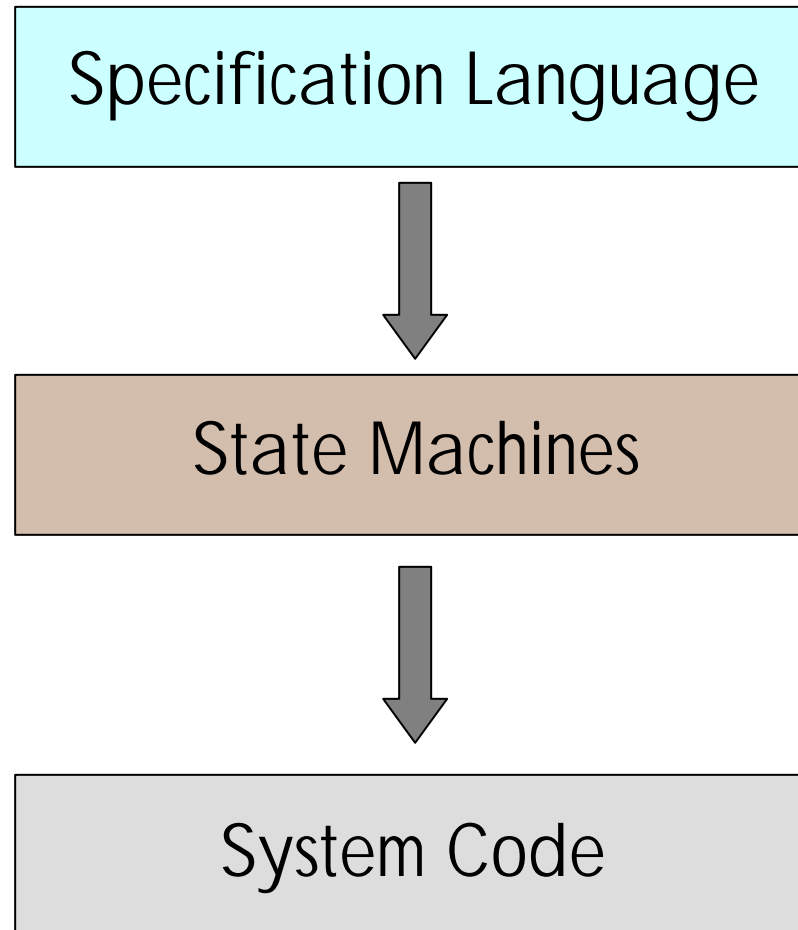
These models support several logics of processes, e.g. temporal logics, programming logics, etc.

Our work has been based on variants of Nancy Lynch's **IO Automata**. We presented some results in this summer school in 1999, with Jason Hickey.

# Stack Model



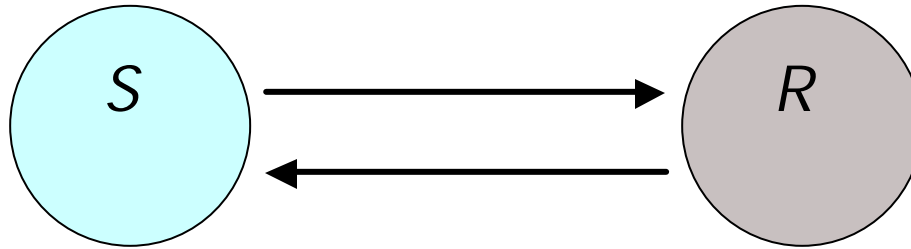
# Event Systems Abstraction



## Event Systems – Key Concepts

- Type  $E$  of **events** – abstract
- Type  $Loc$  of **loci** of events or process identifiers
- Type  $Lnk$  of communication **links**
- There are two **kinds** of events: (1) **local** actions and (2) **receives**; each can also **send** on links
- Each receive event has a unique **sender**
- Events at any locus are **totally ordered**, and **causal order** relates events among loci

## Example – Two-Phase Handshake

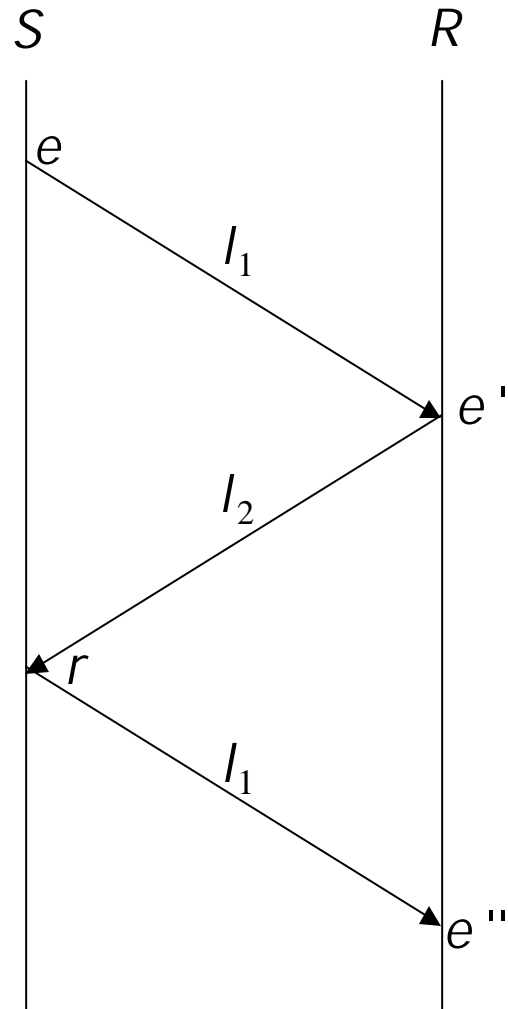


$$E_p = \{e : E \mid loc(e) = p\}$$

$$Snd_{p,l} = \{e : E_p \mid sends(e, l) \neq nul\}$$

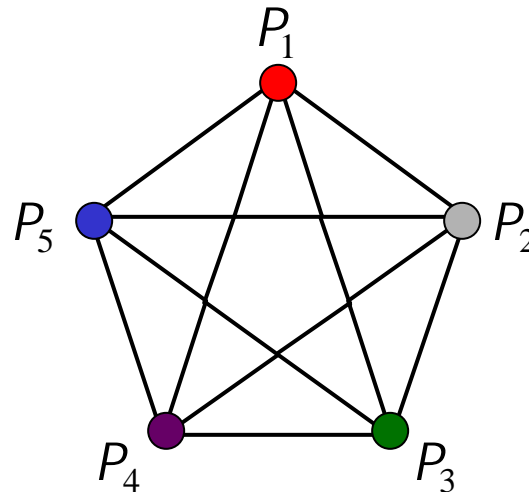
$$Rcv_{p,l} = \{e : E_p \mid kind(e) \text{ is receive on } l\}$$

# Communication Diagram



# Distributed Systems

A distributed system  $D$  will be a graph  $G$  whose nodes are **processes** and whose **links** are communication channels.



We will treat processes abstractly in these lectures. In our verification work we use **message automata** and IO Automata at the nodes. We can also imagine other abstractions – e.g. active objects – as well as CML or Java programs.

# Executions of Distributed Systems

Executions of distributed systems are event systems in a natural way. Executions are typically indexed by time, and that can be discrete, say  $t \in \mathbb{N}$ .

At each moment of time, a process at  $i$  is in a **state**,  $s(i, t)$ , and the links are lists of tagged messages,  $m(l, t)$ . At each locus  $i$  and time  $t$ , there is an action,  $a(i, t)$ , taken. The action can be null, i.e. no state change, no receives, hence no sends.

## Fair-Fifo Executions

We assume executions are **fair**: channels are loss-less; and **fifo**: messages are received in the order sent.

1. Only the process at  $i$  can send messages on links originating at  $i$ .
2. A receive action at  $i$  must be on a link whose destination is  $i$  and whose message is at the head of the queue on that link.
3. There can be **null actions** that leave a state unchanged between  $t$  and  $t + 1$ .
4. Every queue is examined **infinitely often**, and if it is nonempty, a message is delivered.
5. The **precondition** of every local action is examined infinitely often and if true the action is taken.

## Event Systems of Fair-Fifo Executions

If  $w$  is a fair-fifo execution of a distributed system  $D$ , we can define an event system from it,  $Ev(w)$ .

The **types**  $Loc, Lnk, Kind, Tag, Id$  are inherited from  $D$ .

The **events**  $E$  are the points  $\langle i, t \rangle$  locus  $i$  and time  $t$ , at which a non-null action, local or receive, occurred in  $w$ .

# Specifying Protocols and Systems

Function specification:

$$\vdash ?x : A. ?y : B. R(x, y) \text{ ext } f : A \rightarrow B$$

Protocol specification is part of a system specification.

$$\bar{x} : \text{System} \vdash \text{ProcessSpec}(x_i) \text{ ext } P_i$$

We will focus on the *ProcessSpec*.

# Axioms

**Axiom 1:** For every event  $e$  which sends a list of messages on link  $l$ , we can find an event  $e'$  at the destination of  $l$  at which all messages sent are received.

$$?e : E . ?l : Lnk . ?e' : E . ?e'' : E .$$

$$receive(e'') \Rightarrow e = sender(e'') \Rightarrow$$

$$link(e'') = l \Rightarrow (e' = e'' \vee loc(e') = dst(l))$$

**Axiom 2:** The predecessor function at each locus is one-to-one.

$$?e, e' : E . loc(e) = loc(e') \Rightarrow$$

$$(pred(e) = pred(e') \Rightarrow e = e')$$

# Axioms

**Axiom 3 :** The causal order  $<$  is strongly well-founded.

$$\exists f : E \rightarrow \mathbb{N}. \forall e, e' : E. (e < e' \Rightarrow f(e) < f(e')).$$

**Axiom 4 :** If  $e$  is not the first event at a locus, then its predecessor is at the same location.

$$\forall e : E. \neg \text{first}(e) \Rightarrow \text{loc}(\text{pred}(e)) = \text{loc}(e).$$

**Axiom 5 :** If  $e$  is a *receive* event, then the locus of the sender is the source of the link of  $e$ .

$$\forall e : E. \text{receive}(e) \Rightarrow \text{loc}(\text{sender}(e)) = \text{src}(\text{link}(e)).$$

## Axioms

**Axiom 6** : Messages on a link are received in the order sent.

$$\begin{aligned} ?e, e' : E. (rcv(e) \Rightarrow rcv(e')) \Rightarrow \\ (link(e) = link(e')) \Rightarrow \\ (sender(e) < sender(e')) \Rightarrow e < e'. \end{aligned}$$

**Axiom 7** : By convention, for any event except the first, to say that an observable  $x$  has a value  $v$  when event  $e$  happens at the locus of  $e$  is to say that  $x$  after the predecessor of  $e$  is  $v$ .

$$\begin{aligned} ?e : E. \neg first(e) \Rightarrow ?x : Id. \\ (x \text{ when } e = x \text{ after } pred(e)) \end{aligned}$$

## Deriving Algorithms and Protocols

Sequential program derivation from a specification:

$$\vdash \exists p : State \rightarrow State. \ ?s : State. R(s, p(s))$$

Nuprl supports extraction.

$$\vdash \ ?s : State. \ ?s' : State. R(s, s') \text{ ext } p$$

# Refinements for Programs

$\vdash ?x : A. ?y : B. R(x, y) \text{ ext } \mathbf{!}x. \text{cut}(x, z. g(x, z); l(x))$

$x : A \vdash ?y : B. R(x, y) \text{ ext } \text{cut}(x, z. g(x, z); l(x))$

by cut L

1.  $x : A, z : L \vdash ?y : B. R(x, y) \text{ ext } g(x, z)$

by  $D o \ulcorner g(x, z) \urcorner$

$x : A, z : L \vdash R(x, g(x, z))$

2.  $x : A \vdash L \text{ ext } l(x)$

by \_\_\_\_\_

# Refinements for Systems

$\vdash ?D : \text{System}. ?es : ES(D).$

$R(es) \text{ ext } \text{Comp}(pf_1(D_1, es_1), pf_2(D_2, es_2))$

by *Comp*

1.  $D_1 : \text{System}(G, Loc, Lnk)$

$es_1 : ES(D_1) \vdash R_1(es_1) \text{ ext } pf_1(D_1, es_1)$

2.  $D_2 : \text{System}(G, Loc, Lnk)$

$es_2 : ES(D_2) \vdash R_2(es_2) \text{ ext } pf_2(D_2, es_2)$

## Event Systems $es$ of Distributed System $D$

For an (abstract) distributed system  $D$ , say that  $es$  is an **event system of  $D$** ,  $es \in ES(D)$ , if  $es$  is the event system  $Ev(w)$  of an execution  $w$  of  $D$ .

## Deriving the Two-Phase Handshake

We illustrate this process by deriving a protocol for the two-phase handshake from a proof that its specification is **realizable**.

$$(1) \forall e_1, e_2 : Snd_{S,l_1} \cdot ?r : Rcv_{S,l_2} \cdot (e_1 < e_2 \Rightarrow e_1 < r < e_2)$$

$$(2) \forall e_1, e_2 : Snd_{R,l_2} \cdot ?r_1, r_2 : Rcv_{R,l_1} \cdot (e_1 < e_2 \Rightarrow r_1 < e_1 < r_2 < e_2)$$

## Deriving Handshake – 1

One way to achieve alternation of sends and receives at  $S$  is to introduce a **boolean variable  $rdy$**  and stipulate that  $S$  sends only when  $rdy$  is true. When a send on  $l_1$  occurs,  $rdy$  is set to false.

**Lemma 1:** For any two processes  $S, R$ , and links  $l_1, l_2$ ,  
 $src(l_1) = S \quad \& \quad dst(l_1) = R, \quad src(l_2) = R$   
 $\& \quad dst(l_2) = S, \quad \& \quad rdy \text{ in } Id,$

we can realize an event system with the property that initially  $rdy = true$  at  $S$ .

Call the realizer for Lemma 1  $ES_1$ .

## Deriving Handshake – 2

**Lemma 2 :** We can find a realizer that extends  $ES_1$  such that for any non-empty type  $T$ ,

$$\forall e : E_S. (rdy \text{ when } e = true) \Rightarrow$$
$$?e' : E_S. e < e' \wedge (?v : T. (rdy \text{ after } e' = false \wedge$$
$$sends(e', l) = [l_1, val, v]) \vee rdy \text{ when } e' = false).$$

**Proof :**

Infinitely often the process at  $S$  will examine its precondition  $rdy = true$ . If  $rdy$  has not changed, then the send will occur, and  $rdy$  will be set to false. Otherwise,  $rdy$  must be false at  $t$ , so some event  $e'$  after  $e$  set it to false.

**Qed**

Call the realizer for Lemma 2  $ES_2$ .

## Deriving Handshake – 3

**Lemma 3 :** We can find a realizer that extends  $ES_2$  such that

$$\forall e : E_S. \text{ sends } (e, l_1) \neq \text{nil} \Rightarrow \text{rdy after } e = \text{false}.$$

**Proof :**

We constrain the realizer of Lemma 2 to satisfy the **frame condition** that there are no actions that send on  $l_1$  with tag  $val$  except for the one specified in Lemma 2.

**Qed**

Call the realizer for Lemma 3  $ES_3$ .

## Deriving Handshake – 3

**Lemma 4 :** We can find a realizer that extends  $ES_3$  such that

$$\forall e : E_S. \text{rcv}(e, I_2) \Rightarrow \text{rdy after } e = \text{true}.$$

**Proof :**

We add to the process at  $S$  a response to any receive action on  $I_2$ , namely this receive will set  $\text{rdy}$  to true.

**Qed**

Call this realizer  $ES_4$ .

## Deriving Handshake – 4

**Lemma 5 :** We can find realizers that extend  $ES_5$  such that

$$\forall e : E_S. \text{rcv}(e, I_2) \Rightarrow \exists e' : E_S. e < e' \wedge \text{sends}(e, I_1) \neq \text{nil}.$$

**Proof :**

We add the frame condition that only a receive on  $I_2$  or reset of  $rdy$  to *false* can affect  $rdy$ .

Note,  $rdy$  after  $e = \text{true}$  by Lemma 4. Execution is fair, thus the precondition of the send will be checked **infinitely often**, hence after  $e$ . Only a reset can change  $rdy$  to *false* by the frame condition, and this happens only if the send on  $I_1$  is executed. Thus there will be an event  $e'$  after  $e$  such that the send is executed.

**Qed**

Call this realizer  $ES_5$ .

## Deriving Handshake – 4

**Theorem 1:** Any realizer extending  $ES_5$  satisfies

$$\forall e_1, e_2 : Snd_{S, I_1}. (e_1 < e_2 \Rightarrow \exists r : Rcv_{S, I_2}. e_1 < r < e_2)$$

**Proof :**

Let  $e_1, e_2$  be send events at  $S$  on link  $I_1$ . Suppose  $e_1 < e_2$ .  
By Lemma 3,  $rdy$  after  $e_1 = false$ . The only event that can set  $rdy$  to  $true$  is a receive event. the only way  $e_1 < e_2$  is possible is that  $rdy$  when  $e_2 = true$ . Thus before  $e_2$  and after  $e_1$  there must be a receive event.

**Qed**

# Lamport's TLA+ Specification – 2003

EXTENDS *Naturals* CONSTANT *Data*

VARIABLES *val*, *rdy*, *ack*

*TypeInvariant*  $\triangleq \wedge val ? Data \wedge rdy ? \{0,1\} \wedge ack ? \{0,1\}$

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*Init*  $\triangleq val ? Data \wedge rdy ? \{0,1\} \wedge ack = rdy$

*Send*  $\triangleq \wedge rdy = ack$   
 $\wedge val' ? Data$   
 $\wedge rdy' = 1 - rdy$   
 $\wedge \text{UNCHANGED } ack$

*Rcv*  $\triangleq \wedge rdy \neq ack$   
 $\wedge ack' = 1 - ack$   
 $\wedge \text{UNCHANGED } \langle val, rdy \rangle$

*Next*  $\triangleq Send \vee Rcv$

*Spec*  $\triangleq \text{Init} \wedge \square [Next]_{\langle val, rdy, ack \rangle}$

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THEOREM  $Spec \Rightarrow \square TypeInvariant$