MetaPRL – A Modular Logical Environment

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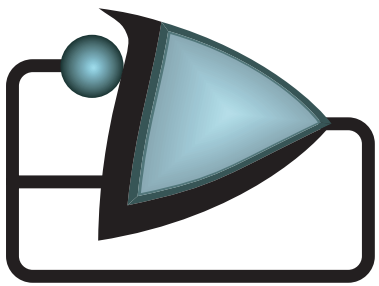
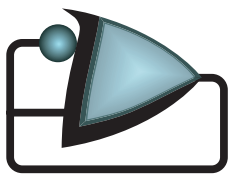
Caltech, Cornell, CUNY, Moscow State, Newton Labs, …



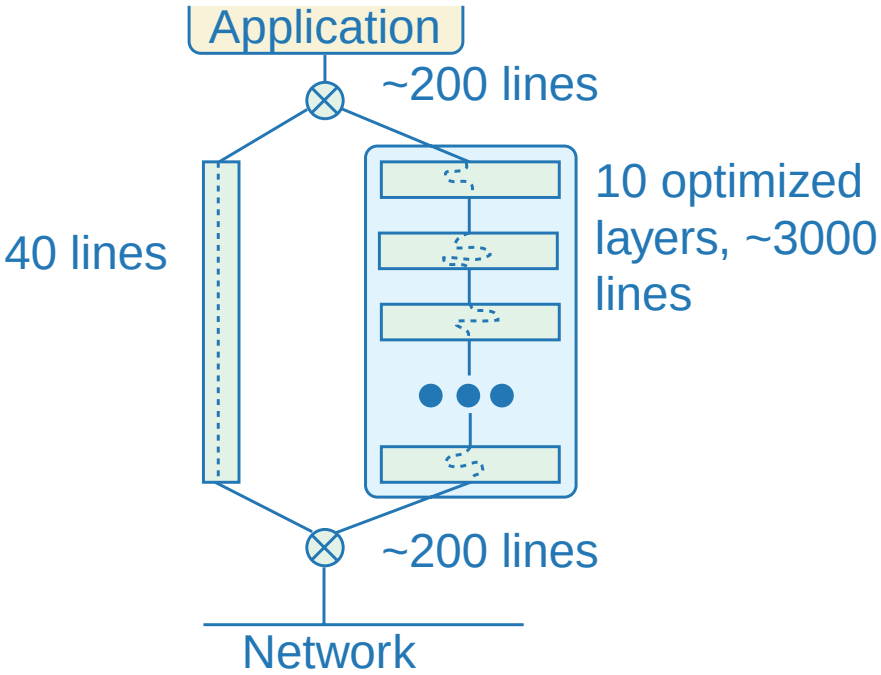
PRL (Proof/Program Refinement Logic) History

- 1970-80s: PRL, Lambda-PRL, NuPRL-3 (Constable et.al.)
- 1992: NuPRL-4
  - *Structured term editing and proof development environment*
  - *Extensive tactic collection and rewrite package (Paul Jackson)*
  - *Logic is the NuPRL type theory, a Martin-LÖf style type theory with many extensions (quotients, recursive types, …)*
- 1998: NuPRL-5
  - *Multi-prover environment*
  - *Extended term/proof editor*
  - *Formal Digital Library for sharing formal knowledge among provers (currently supports PVS,Isabelle, …)*
- Architecture: Common Lisp, Classic ML



1990s: focus on scalability

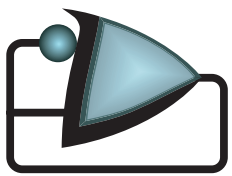
- 1995: Application to the Ensemble Group Communication System (Kreitz, Hayden, Hickey)



Protocols are composed of about 10-30 layers

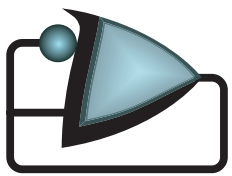
Each layer is about 100-300 lines of OCaml code

NuPRL is used both for verification and optimization (order of magnitude speedup)

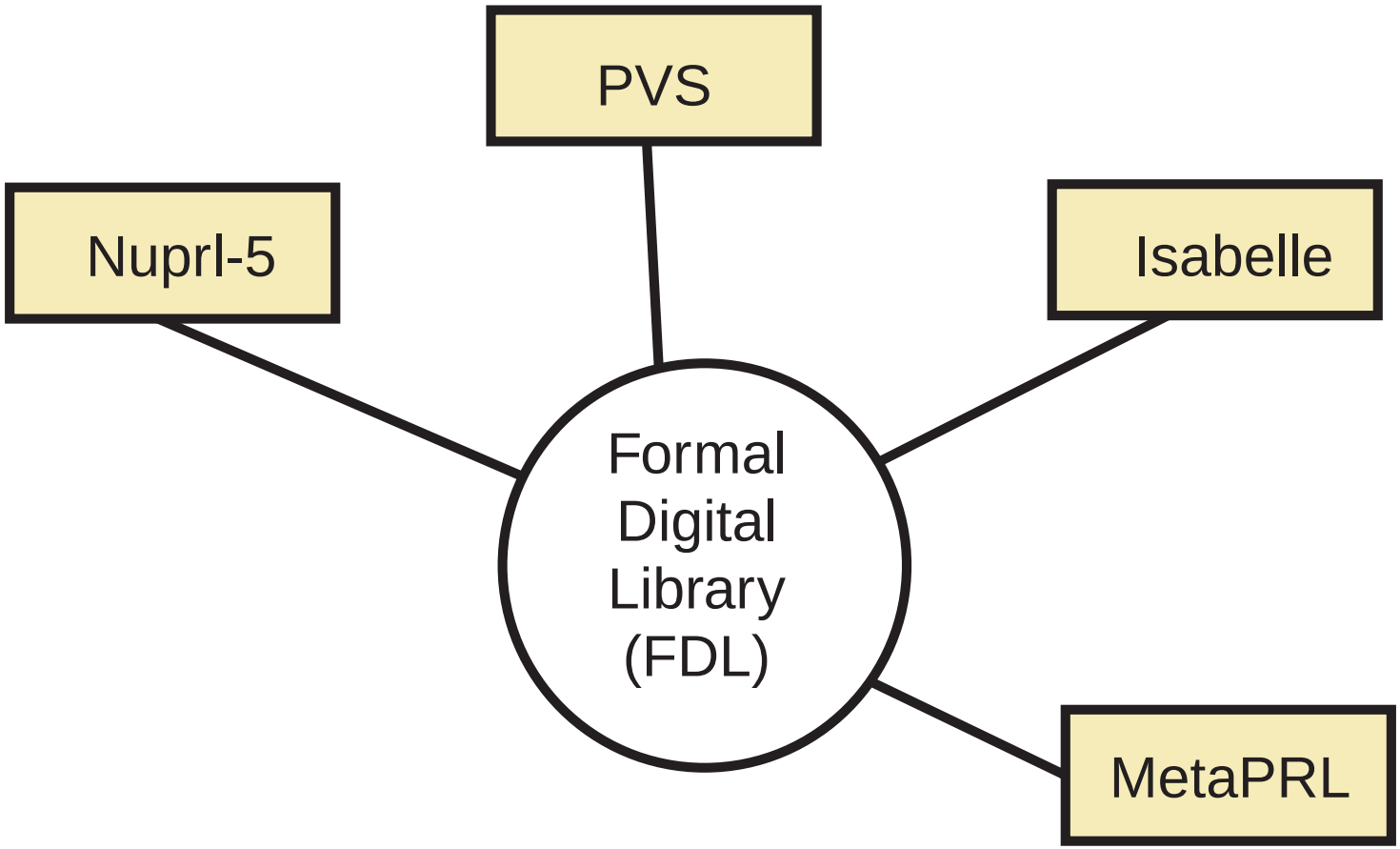


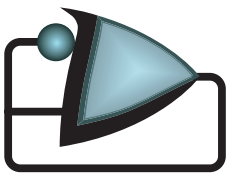
MetaPRL history

- 1995: developed to address problems of scalability (Hickey, NuPRL-Light)
  - *Modular architecture*
    - *Clean separation of trusted logic core from logical* theories
  - *Logical framework and programming environment implemented as an extension to the OCaml compiler (Leroy et.al.)*
  - *Preserves*
    - *NuPRL term structure*
    - *Includes NuPRL type theory as one of the logics*
  - *Adds*
    - *Second-order term matching, rewriting, and refinement*

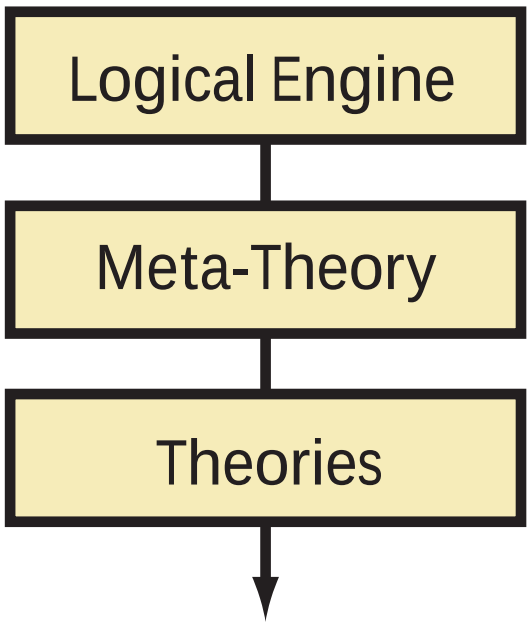


Formal Digital Library

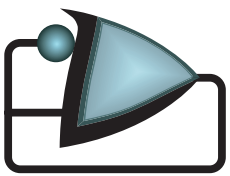




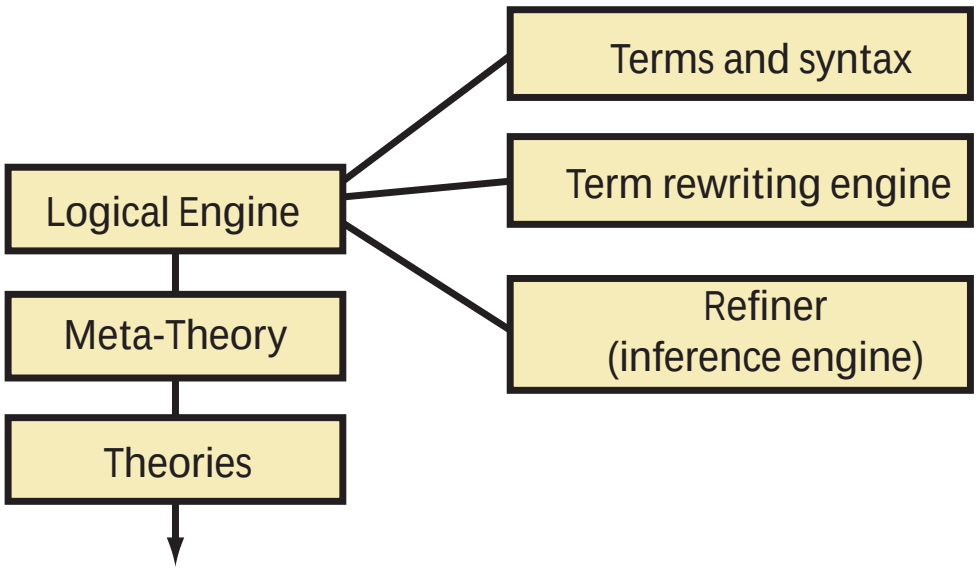
Architecture



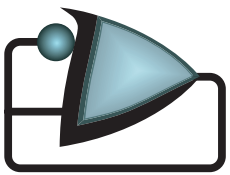
- Logical Engine
  - *defines syntax, rewriting, inference, accounting, checking*
- Meta-theory
  - *Defines a language (OCaml) for directing proofs with tactics and conversions*
- Theories
  - *Logics, type theories, set theories, etc.*
- Note: the user-interface is not an explicit part (though the system provides one)



Architecture II



- Term module
  - *Defines syntax, with binding*
- Rewriting
  - *Defines rewriting with second-order variables and patterns*
- Refiner
  - *Defines inference, accounting, proof management*



An example inference rule

$$\frac{\Gamma, \forall x{:}A.B[x] \vdash a \in A \quad \Gamma, \forall x{:}A.B[x], B[a] \vdash C}{\Gamma, \forall x{:}A.B[x] \vdash C}\ \text{all\_elim}(a)$$

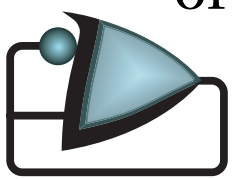
- The rule declaration defines a tactic:

  val all_elim : term → tactic

- The low-level refiner defines an application:

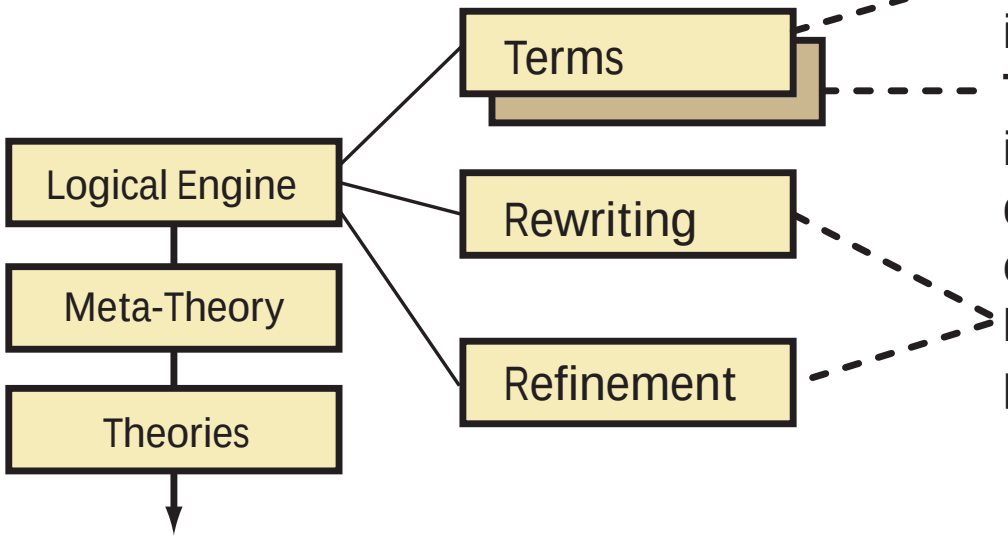
  val refine : tactic → goal → goal list * justification

- The meta-logic uses second-order Horn clauses with second-order meta-variables

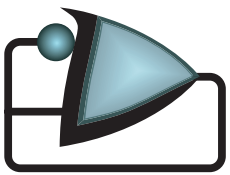


Speed and optimization

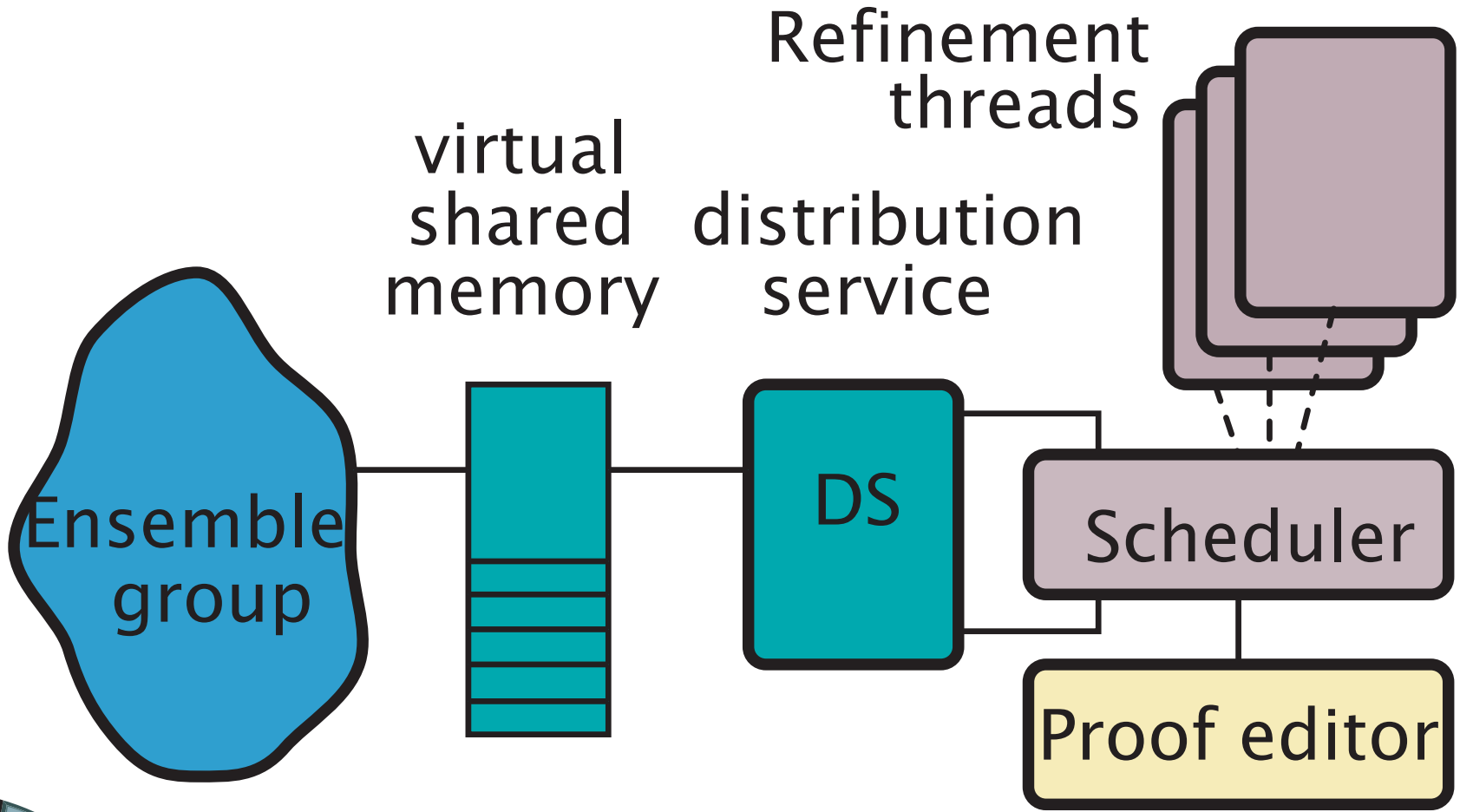
Speed comparison: MetaPRL is consistently x100 faster than NuPRL
-- OCaml vs.Common Lisp/Classic ML (about x5 speed improvement)
-- Highly optimized algorithms and data structures (asymptotic improvement)



Term_std: standard, reference implementation of term
Term_ds: highly optimized implementation of terms with delayed substitution and cached free vars computation
Rewriter: rules and rewrites are precompiled into bytecode programs

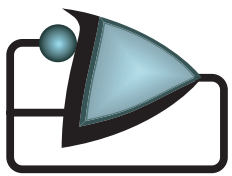


Transparent, fault-tolerant distributed proving



Two main logical concepts

Syntax	`declare` $A \Rightarrow B$ `declare` $\lambda x.b[x]$ `declare` $apply\{f;a\}$
Rewrites	`prim_rw beta:` $apply\{\lambda x.b[x];a\} \longleftrightarrow b[a]$
Rules	`prim imp_intro :` $b[x] : (\Gamma, x{:}A \vdash B) \longrightarrow$ $\Gamma \vdash A \Rightarrow B$ $= \lambda x.b[x]$

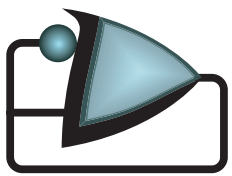


Derived rules (and abstraction)

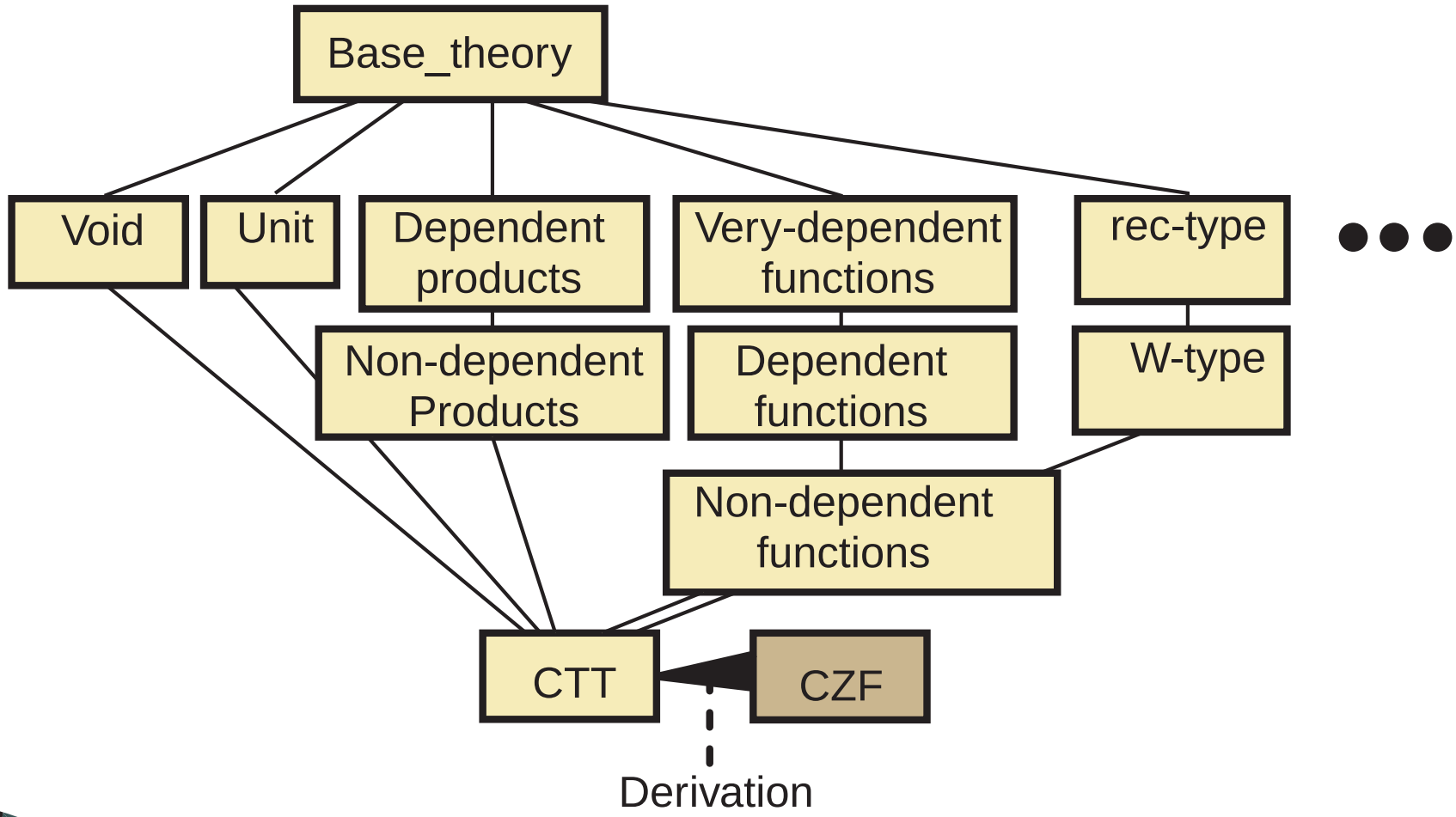
define btree = · · · (μ-type, W-type, etc.)
define leaf = · · ·
define node{a; b} = · · ·

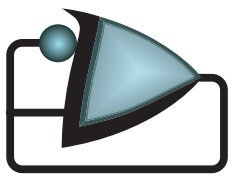
interactive btree_ind :

$$\frac{\Gamma \vdash P[leaf] \quad \Gamma, a: btree, P[a], b: btree, P[b] \vdash P[node(a, b)]}{\Gamma, x: btree \vdash P[x]}$$



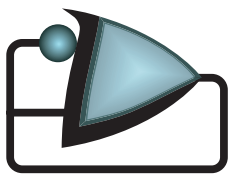
Theory hierarchy





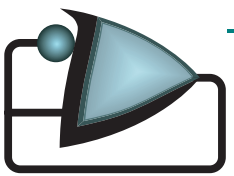
Resources

- Context-sensitive proof automation
- Generic tactics
  - *dT : generic application of introduction and elimination reasoning*
  - *autoT : generic automated reasoning*
- The behavior of the tactic depends on the context
- Resources collect components for generic, context-sensitive, data structures (including dT, autoT)



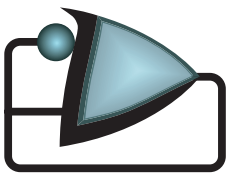
Theories in MetaPRL

- CTT (Computational Type Theory)
  - *Based on the NuPRL type theory (a Martin-Löf style type theory)*
  - *Very-dependent function types (Hickey)*
  - *New,modular account of quotient types (Nogin)*
  - *Extensible dependent records (Kopylov)*
- CZF (Aczel's Constructive ZF set theory)
  - *Fully axiomatized and derived from CTT*
- Programming language logics
  - *A theory of imperative programs with operational semantics*
  - *Theories of compilers and intermediate languages*
  - *…*



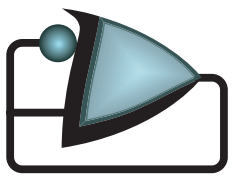
Tactics and decision procedures

- NuPRL implemented arithmetic as a *trusted* decision procedure
  - *MetaPRL axiomatizes arithmetic*
  - *The arith decision procedure is a tactic that produces a proof derived from the axioms (Brukhoff, Nogin, TPHOLs 2003)*
    - *Note Norish's work on adding decision procedures to HOL (TPHOLs 2003)*
- JProver
  - *A highly-optimized first-order prover that produces a proof to be checked in MetaPRL (Kreitz, Schmitt)*



Other ongoing work

- Compilers (e.g. Hickey et.al. Merlin 2003)
  - *A complete compiler for a small ML-like language*
  - *CPS, closure conversion, optimization, code generation*
  - *The compiler is implemented as a set of rewrites (using HOAS)*
  - *The trusted core is about 200 rewrites*
  - *Generates functional(!) assembly code for the Intel x86 platform*
  - *Extremely easy to implement because of all the usual things: built-in substitution, generic rewriting strategies, perservation of scoping, etc…*



Summary

- MetaPRL is designed to be fast, flexible, modular
  - *In part, MetaPRL is a logical toolkit for formal methods (terms, rewriting, decision procedures, etc)*
  - *It serves as part of the FDL, sharing proofs with systems including Nuprl, PVS, Isabelle, with planned support for others like Coq, HOL, Larch, etc*
- It is a logical framework
  - *Emphasizing organization and relation of logics*
- It is designed as a logical programming environment
  - *Clean integration with the OCaml programming language*
- It is open-source, freely available
  - *See metaprl.org*