Order-theoretic Differences Between Two Variants of Type Theory

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Abstract

Two versions of computational type theory under active development around town are Nuprl 5 and a (still-unnamed) variant of it that arises from Howe's classical, set-theoretic semantics. This intermediate-level seminar will compare an aspect of their related but different subtyping orders.

In particular, as a way to tour a bit of the crisp and flourishing structure present in these systems, we will consider the question of whether the universe-spanning intersection type $\bigcap_{T:U_1} T \to T$ is atomic in the order-theoretic sense—i.e., whether it has, modulo extensional type equality, any subtypes other than itself and the empty one. It will turn out that the intuitive, hand-waving argument based on predicativity actually yields an invalid conclusion for the radically constructive context, whereas in the classical one it not only gives the right answer but also matches the high-level flow of the proof.

If there's time, I will additionally briefly sketch out an alternative construction for the latter that uses separativeness instead of rank. This has the advantage of not bumping up a universe level in the generalized case, $\bigcap_{i:I} T_i \to T_i'$.

A secondary theme will be small illustrations of how all the experience you've gained over the years while working with order theory—especially with lattices and with equivalence relations—will typically serve you well in this initially unfamiliar domain.