```
eval_factorization_pluck  THM

⊢  ∀a,b:ℤ, f:({a..b⁻}→ℕ), z:{a..b⁻}.
|  0<f(z) ⇒ Π↑{a..b⁻}(f) = z·Π↑{a..b⁻}(reduce_factorization(f; z))
| by Auto
|
1. a : ℤ
2. b : ℤ
3. f : {a..b⁻}→ℕ
4. z : {a..b⁻}
5. 0<f(z)
⊢  Π↑{a..b⁻}(f) = z·Π↑{a..b⁻}(reduce_factorization(f; z))
| by Let (X = reduce_factorization(f; z))
|     THEN
|     Rewrite← by
|     Thm*  ∀a,b:ℤ, j:{a..b⁻}. Π↑{a..b⁻}(trivial_factorization(j)) = j
|     {eval_trivial_factorization}
|     Using:[j:= z]
|     THEN
|     Rewrite by Hyp:7
|
6. X : {a..b⁻}→ℕ
7. X = reduce_factorization(f; z)
⊢  Π↑{a..b⁻}(f)
|  =
|  Π↑{a..b⁻}(trivial_factorization(z))
|  ·Π↑{a..b⁻}(reduce_factorization(f; z))
| by Rewrite by
|     Thm*  ∀a,b:ℤ, f,g:({a..b⁻}→ℕ).
|           Π↑{a..b⁻}(f)·Π↑{a..b⁻}(g) = Π↑{a..b⁻}(λi.f(i)+g(i))
|     {eval_factorization_prod}
|
⊢  Π↑{a..b⁻}(f)
|  =
|  Π↑{a..b⁻}(λi.trivial_factorization(z)(i)+reduce_factorization(f; z)
|                                         (i))  by Analyze
|
⊢  f
|  =
|  (λi.trivial_factorization(z)(i)+reduce_factorization(f; z)(i))
|  ∈ {a..b⁻}→ℤ  by FunExtensionality
|
8. x : {a..b⁻}
⊢  f(x) = trivial_factorization(z)(x)+reduce_factorization(f; z)(x)
| by Def of trivial_factorization(z) | reduce_factorization(f; z)
|     THEN
|     Reduce Concl
|
⊢  f(x) = if x=₂z→ 1 else 0 fi+if x=₂z→ f(x)-1 else f(x) fi
  by SplitOnConclITEs
END
```