```
eval_reduce_factorization_less  THM

⊢  ∀a:ℕ⁺, b:ℤ, f:({a..b⁻}→ℕ), j:{a..b⁻}.
|  2≤j ⇒ 0<f(j) ⇒ Π↑{a..b⁻}(reduce_factorization(f; j))<Π↑{a..b⁻}(f)
| by Autoℕ
|
1. a : ℕ⁺
2. b : ℤ
3. f : {a..b⁻}→ℕ
4. j : {a..b⁻}
5. 2≤j
6. 0<f(j)
⊢  Π↑{a..b⁻}(reduce_factorization(f; j))<Π↑{a..b⁻}(f)
| by Rewrite by
|    Thm*  ∀a,b:ℤ, f:({a..b⁻}→ℕ), z:{a..b⁻}.
|          0<f(z)
|          ⇒
|          Π↑{a..b⁻}(f) = z·Π↑{a..b⁻}(reduce_factorization(f; z))
|    {eval_factorization_pluck}
|    Using:[z:= j] ...ℕ
|
⊢  Π↑{a..b⁻}(reduce_factorization
|            (f; j))<j·Π↑{a..b⁻}(reduce_factorization(f; j))
| by BackThru:
|    Thm*  ∀i:{2...}, j:{1...}. j<i·j    {factor_smaller} ...ℕ
|
.....wf .....
⊢  Π↑{a..b⁻}(reduce_factorization(f; j)) ∈ {1...}
  by BackThru:
     Thm*  ∀a:ℕ⁺, b:ℤ, f:({a..b⁻}→ℕ). Π↑{a..b⁻}(f) ∈ ℕ⁺
     {eval_factorization_nat_plus} ...ℕ
END
```