```
nat_prime_div_each_factorLEMMA  THM

⊢  ∀X:ℕ.
|  prime(X)
|  ⇒
|  (∀X1:ℕ. X1<X ⇒ prime(X1) ⇒ (∀a,b:ℕ. X1 | a·b ⇒ X1 | a ∨ X1 | b))
|  ⇒
|  (∀W:ℕ. 0<W ⇒ W<X ⇒ (∀t:ℕ. X | t·W ⇒ X | t))
| by Autoℕ THEN RemoveGuards Concl ...wℕ
|
1. X : ℕ
2. prime(X)
3. ∀X1:ℕ. X1<X ⇒ prime(X1) ⇒ (∀a,b:ℕ. X1 | a·b ⇒ X1 | a ∨ X1 | b)
⊢  ∀W:ℕ. 0<W ⇒ W<X ⇒ (∀t:ℕ. X | t·W ⇒ X | t)
| by CompNatInd Concl ...ℕ
|
4. W : ℕ
5. ∀W1:ℕ. W1<W ⇒ 0<W1 ⇒ W1<X ⇒ (∀t:ℕ. X | t·W1 ⇒ X | t)
6. 0<W
7. W<X
8. t : ℕ
9. X | t·W
⊢  X | t
| by Rewrite'-1 by
|    Thm*  ∀a,b:ℕ. a | b ⟺ (∃c:ℕ. b = a·c)
|    {divides_def_on_nat} ...ℕ
|    THEN
|    New:u Analyze-1
|
9. u : ℕ
10. t·W = X·u
⊢  X | t  by Decide: W = 1 ...wℕ
|
|\
| 11. W = 1
| ⊢  X | t  by Rewrite10 by W = 1 ...ℕ THEN Witness: u ...ℕ
|
.\
. 11. ¬W = 1
. ⊢  X | t
. | by Inst:
. |    Thm*  ∀x:{2...}. ∃p:{2...}. p≤x & prime(p) & p | x
. |    {prime_or_smaller_prime_factor}
. |    Using:[W] ...wℕ
. |    THEN
. |    New:X1 (ExistHD Hyp:-1)
. |
. 12. X1 : {2...}
. 13. X1≤W
. 14. prime(X1)
. 15. X1 | W
. ⊢  X | t
. | by Rewrite'-1 by
. |    Thm*  ∀a,b:ℕ. a | b ⟺ (∃c:ℕ. b = a·c)
. |    {divides_def_on_nat} ...wℕ
```

```
. |    THEN
. |    Analyze-1
. |
. 15. c : ℕ
. 16. W = X1·c
. ⊢  X | t  by Rewrite10 by W = X1·c ...ℕ
. |
. 17. t·X1·c = X·u
. ⊢  X | t  by Inst: Hyp:3 Using:[X1 | X | u] ...aℕ
. |
. |\
. | .....antecedent .....
. | ⊢  X1 | X·u  by Witness: t·c ...ℕ
. |
. .\
. . 18. X1 | X ∨ X1 | u
. . ⊢  X | t
. . | by FwdThru:
. . |    Thm*  P ∨ Q ⇒ ¬P ⇒ Q    {disjunct_elim}
. . |    on [ Hyp:-1 ] ...ℕ
. . |
. . |\
. . | .....antecedent .....
. . | ⊢  ¬X1 | X
. . |   by BackThru:
. . |      Thm*  ∀x:ℕ. prime(x) ⇒ (∀i:{2..x⁻}. ¬i | x)
. . |      {natprime_nondivs} ...ℕ
. . |
. . .\
. . . 19. X1 | u
. . . ⊢  X | t
. . . | by Rewrite-1 by
. . . |    Thm*  ∀a,b:ℕ. a | b ⟺ (∃c:ℕ. b = a·c)
. . . |    {divides_def_on_nat} ...wℕ
. . . |    THEN
. . . |    New:d Analyze-1
. . . |
. . . 20. d : ℕ
. . . 21. u = X1·d
. . . ⊢  X | t
. . . | by Rewrite17 by Hyp:-1 ...wℕ
. . . |    THEN
. . . |    MulFactorOut [] Hyp:-1 ...wℕ
. . . |
. . . 22. t·c = X·d
. . . ⊢  X | t  by Decide: c = 1 ...wℕ
. . . |

. . . |\
. . . | 23. c = 1
. . . | ⊢  X | t  by Witness: d ...ℕ THEN Rewrite22 by c = 1 ...ℕ
. . . |
. . . .\
. . . . 23. ¬c = 1
. . . . ⊢  X | t
. . . . | by 0<c | c<W  Asserted
```

```
. . . . |    THEN
. . . . |    (Inst: Hyp:5 Using:[c | t] ...ℕ)
. . . . |
. . . . |\
. . . . | .....assertion .....
. . . . | ⊢  0<c
. . . . |   by FwdThru:
. . . . |      Thm*  ∀a,b:ℕ. 0<a·b ⟺ 0<a & 0<b
. . . . |      {pos_mul_arg_boundsNat}
. . . . |      on [ 0<c·X1 ] ...ℕ
. . . . |
. . . . |\
. . . . | .....assertion .....
. . . . | 24. 0<c
. . . . | ⊢  c<W
. . . . |   by Inst:
. . . . |      Thm*  ∀i:{2...}, j:{1...}. j<i·j    {factor_smaller}
. . . . |      Using:[X1 | c] ...ℕ
. . . . |
. . . . .\
. . . . . .....antecedent .....
. . . . . 24. 0<c
. . . . . 25. c<W
. . . . . ⊢  X | t·c  by Witness: d ...ℕ
END
```