```
only_one_factored_by  THM

⊢  ∀f:(Primeℕ→ℕ), n,n':ℕ.
|  f is a factorization of n ⇒ f is a factorization of n' ⇒ n = n'
| by Analyze
|
1. f : Primeℕ→ℕ
⊢  ∀n,n':ℕ.
|  f is a factorization of n ⇒ f is a factorization of n' ⇒ n = n'
| by {WLOG assume n≤n' } SideProof 1
|
2. n : ℕ
3. n' : ℕ
4. n≤n'
5. f is a factorization of n
6. f is a factorization of n'
⊢  n = n'  by Analyze-1 THEN Analyze5
|
7. ∀x:Primeℕ. n'<x ⇒ f(x) = 0
8. n' = Π↑{2..n'+1⁻}(prime_mset_complete(f))
9. ∀x:Primeℕ. n<x ⇒ f(x) = 0
10. n = Π↑{2..n+1⁻}(prime_mset_complete(f))
⊢  n = n'  by Rewrite by Hyp:-1 THEN Rewrite by Hyp:8
|
⊢  Π↑{2..n+1⁻}(prime_mset_complete(f))
|  =
|  Π↑{2..n'+1⁻}(prime_mset_complete(f))
| by FwdThru:
|    Thm*  ∀f:(Primeℕ→ℕ), n:ℕ. f is a factorization of n ⇒ 0<n
|    {only_positives_prime_fed}
|    on [ Hyp:5 ]
|
11. 0<n
⊢  Π↑{2..n+1⁻}(prime_mset_complete(f))
|  =
|  Π↑{2..n'+1⁻}(prime_mset_complete(f))
| by Rewrite by
|    Thm*  ∀a,c,b:ℤ, f:({a..b⁻}→ℤ).
|           a≤c ⇒ c≤b ⇒ Π↑{a..b⁻}(f) = Π↑{a..c⁻}(f)·Π↑{c..b⁻}(f)
|    {eval_factorization_split_mid}
|    Using:[a:= 2 | b:= n'+1 | c:= n+1 | f:= prime_mset_complete(f)]
|
⊢  Π↑{2..n+1⁻}(prime_mset_complete(f))
|  =
|  Π↑{2..n+1⁻}(prime_mset_complete(f))
|  ·Π↑{n+1..n'+1⁻}(prime_mset_complete(f))
| by Rewrite by Π↑{n+1..n'+1⁻}(prime_mset_complete(f)) = 1
|
.....assertion .....
⊢  Π↑{n+1..n'+1⁻}(prime_mset_complete(f)) = 1
| by BackThru:
|    Thm*  ∀a:{2...}, b:ℤ, f:({a..b⁻}→ℕ).
|           Π↑{a..b⁻}(f) = 1 ⟺ (∀i:{a..b⁻}. f(i) = 0)
|    {eval_factorization_one}
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12. i : {(n+1)..(n'+1)^-}
⊢  prime_mset_complete(f)(i) = 0  by Decide: prime(i)
|
|\
| 13. prime(i)
| ⊢  prime_mset_complete(f)(i) = 0
| | by Rewrite by
| |    Thm*  ∀f:(Primeℕ→ℕ), x:Primeℕ.
| |          prime_mset_complete(f)(x) = f(x)
| |    {prime_mset_complete_isext}
| |
| ⊢  f(i) = 0  by BackThru: Hyp:9
|
.\
. 13. ¬prime(i)
. ⊢  prime_mset_complete(f)(i) = 0
.   by BackThru:
.      Thm*  ∀f:(Primeℕ→ℕ), x:ℕ.
.            ¬prime(x) ⇒ prime_mset_complete(f)(x) = 0
.      {prime_mset_complete_ismin}
END
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only_one_factored_by Side Proof 1  THM

1. f : Primeℕ→ℕ
⊢  ∀n,n':ℕ.
|  f is a factorization of n ⇒ f is a factorization of n' ⇒ n = n'
| by ∀n,n':ℕ.
|    n≤n'
|    ⇒
|    f is a factorization of n ⇒ f is a factorization of n' ⇒ n = n'
|    Asserted
|
|\
| 2. n : ℕ
| 3. n' : ℕ
| 4. n≤n'
| 5. f is a factorization of n
| 6. f is a factorization of n'
| ⊢  n = n'  by <Unproved PREMISE>
|
.\
. 2. ∀n,n':ℕ.
.    n≤n'
.    ⇒
.    f is a factorization of n ⇒ f is a factorization of n' ⇒ n = n'
. 3. n : ℕ
. 4. n' : ℕ
. 5. f is a factorization of n
. 6. f is a factorization of n'
. ⊢  n = n'  by Cases [n≤n' | n'≤n] THEN FwdThru: Hyp:2 on [ Hyp:-1 ]
END
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