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prime_divs_mul_via_intseg  THM

⊢  ∀p:ℕ.
|  prime(p)
|  ⇒
|  (∀a,b:ℤ, e:({a..b⁻}→ℕ).
|   a<b ⇒ p | (Π i:{a..b⁻}. e(i)) ⇒ (∃i:{a..b⁻}. p | e(i)))
| by Guarding (∀b:<type>. <prop>) Autoℕ
|
1. p : ℕ
2. prime(p)
3. a : ℤ
⊢  ∀b:ℤ, e:({a..b⁻}→ℕ).
|  a<b ⇒ p | (Π i:{a..b⁻}. e(i)) ⇒ (∃i:{a..b⁻}. p | e(i))
| by {Use induction over size b-a of the index set } SideProof 1
|
.....assertion .....
⊢  ∀k:ℕ, b:ℤ.
|  b-a = k
|  ⇒
|  (∀e:({a..b⁻}→ℕ).
|   a<b ⇒ p | (Π i:{a..b⁻}. e(i)) ⇒ (∃i:{a..b⁻}. p | e(i)))
| by CompNatInd Concl ...ℕ
|
4. k : ℕ
5. ∀k1:ℕ.
|  k1<k
|  ⇒
|  (∀b:ℤ.
|   b-a = k1
|   ⇒
|   (∀e:({a..b⁻}→ℕ).
|    a<b ⇒ p | (Π i:{a..b⁻}. e(i)) ⇒ (∃i:{a..b⁻}. p | e(i))))
6. b : ℤ
7. b-a = k
8. e : {a..b⁻}→ℕ
9. a<b
10. p | (Π i:{a..b⁻}. e(i))
⊢  ∃i:{a..b⁻}. p | e(i)  by Decide: a+1 = b ...wℕ
|
|\
| 11. a+1 = b
| ⊢  ∃i:{a..b⁻}. p | e(i)
| | by Rewrite10 by
| |    Thm*  ∀a,b:ℤ, e:({a..b⁻}→ℤ).
| |          a+1 = b ⇒ (Π i:{a..b⁻}. e(i)) = e(a)
| |    {mul_via_intseg_singleton}
| |
| 12. p | e(a)
| ⊢  ∃i:{a..b⁻}. p | e(i)  by Witness: a ...ℕ
|
.\
. 11. ¬a+1 = b
. ⊢  ∃i:{a..b⁻}. p | e(i)
. | by Rewrite10 by
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. |    Thm*  ∀a,b:ℤ, e:({a..b⁻}→ℤ).
. |          a<b ⇒ (Π i:{a..b⁻}. e(i)) = (Π i:{a..b-1⁻}. e(i))·e(b-1)
. |    {mul_via_intseg_split_last} ...ℕ
. |
. 12. p | (Π i:{a..b-1⁻}. e(i))·e(b-1)
. ⊢  ∃i:{a..b⁻}. p | e(i)
. | by FwdThru:
. |    Thm*  ∀X:ℕ. prime(X) ⇒ (∀a,b:ℕ. X | a·b ⇒ X | a ∨ X | b)
. |    {nat_prime_div_each_factor}
. |    on [ Hyp:-1 ] ...ℕ
. |    THEN
. |    Analyze-1
. |
. |\
. | 13. p | (Π i:{a..b-1⁻}. e(i))
. | ⊢  ∃i:{a..b⁻}. p | e(i)
. | | by FwdThru: Hyp:5 on [ Hyp:-1 ] Using:[k-1] ...ℕ
. | |
. | 14. ∃i:{a..(b-1)⁻}. p | e(i)
. | ⊢  ∃i:{a..b⁻}. p | e(i)  by SimilarTo Hyp:-1 ...ℕ
. |
. .\
. . 13. p | e(b-1)
. . ⊢  ∃i:{a..b⁻}. p | e(i)  by Witness: b-1 ...ℕ
END
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prime_divs_mul_via_intseg Side Proof 1  THM

1. p : ℕ
2. prime(p)
3. a : ℤ
⊢  ∀b:ℤ, e:({a..b⁻}→ℕ).
|  a<b ⇒ p | (Π i:{a..b⁻}. e(i)) ⇒ (∃i:{a..b⁻}. p | e(i))
| by {Use induction over size of the index set }
|    ∀k:ℕ, b:ℤ.
|    b-a = k ∈ ℤ
|    ⇒
|    (∀e:({a..b⁻}→ℕ).
|     a<b ⇒ p | (Π i:{a..b⁻}. e(i)) ⇒ (∃i:{a..b⁻}. p | e(i)))
|    Asserted
|
|\
| .....assertion .....
| ⊢  ∀k:ℕ, b:ℤ.
|    b-a = k
|    ⇒
|    (∀e:({a..b⁻}→ℕ).
|     a<b ⇒ p | (Π i:{a..b⁻}. e(i)) ⇒ (∃i:{a..b⁻}. p | e(i)))
|   by <Unproved PREMISE>
|
.\
. 4. ∀k:ℕ, b:ℤ.
. |  b-a = k
. |  ⇒
. |  (∀e:({a..b⁻}→ℕ).
. |   a<b ⇒ p | (Π i:{a..b⁻}. e(i)) ⇒ (∃i:{a..b⁻}. p | e(i)))
. ⊢  ∀b:ℤ, e:({a..b⁻}→ℕ).
. |  a<b ⇒ p | (Π i:{a..b⁻}. e(i)) ⇒ (∃i:{a..b⁻}. p | e(i))
. | by Autoℕ
. |
. 5. b : ℤ
. 6. e : {a..b⁻}→ℕ
. 7. a<b
. 8. p | (Π i:{a..b⁻}. e(i))
. ⊢  ∃i:{a..b⁻}. p | e(i)  by BackThru: Hyp:4 Using:[b-a] ...ℕ
END
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