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prime_factorization_exists  THM

⊢  ∀n:{1...}.
|  ∃h:({2..(n+1)⁻}→ℕ).
|  n = Π↑{2..n+1⁻}(h) & is_prime_factorization(2; (n+1); h)  by Autoℕ
|
1. n : {1...}
⊢  ∃h:({2..(n+1)⁻}→ℕ).
|  n = Π↑{2..n+1⁻}(h) & is_prime_factorization(2; (n+1); h)
| by ∃g:({2..(n+1)⁻}→ℕ). n = Π↑{2..n+1⁻}(g)  Asserted
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| .....assertion .....
| ⊢  ∃g:({2..(n+1)⁻}→ℕ). n = Π↑{2..n+1⁻}(g)  by Decide: n = 1
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| | 2. n = 1
| | ⊢  ∃g:({2..(n+1)⁻}→ℕ). n = Π↑{2..n+1⁻}(g)
| |   by {empty factorization }
| |      Rewrite by n+1 = 2 ∈ ℕ
| |      THEN
| |      Witness: λi.i THEN Compute ⌈Π↑{2..2⁻}(λi.i)⌉ *⇒ 1
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| .\
| . 2. ¬n = 1
| . ⊢  ∃g:({2..(n+1)⁻}→ℕ). n = Π↑{2..n+1⁻}(g)
| .   by {just factor n as itself }
| .      Witness: trivial_factorization(n) ...ℕ
| .      THEN
| .      BackThru:
| .      Thm*  ∀a,b:ℤ, j:{a..b⁻}.
| .             Π↑{a..b⁻}(trivial_factorization(j)) = j
| .      {eval_trivial_factorization} ...wℕ
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. 2. ∃g:({2..(n+1)⁻}→ℕ). n = Π↑{2..n+1⁻}(g)
. ⊢  ∃h:({2..(n+1)⁻}→ℕ).
. |  n = Π↑{2..n+1⁻}(h) & is_prime_factorization(2; (n+1); h)
. | by Analyze-1
. |
. 2. g : {2..(n+1)⁻}→ℕ
. 3. n = Π↑{2..n+1⁻}(g)
. ⊢  ∃h:({2..(n+1)⁻}→ℕ).
. |  n = Π↑{2..n+1⁻}(h) & is_prime_factorization(2; (n+1); h)
. | by Inst:
. |    Thm*  ∀k:{2...}, n:ℕ, g:({2..k⁻}→ℕ).
. |          2 ≤ n < k+1
. |          ⇒
. |          (∀i:{2..k⁻}. n≤i ⇒ 0<g(i) ⇒ prime(i))
. |          ⇒
. |          (∃h:({2..k⁻}→ℕ).
. |           Π↑{2..k⁻}(g) = Π↑{2..k⁻}(h)
. |           & is_prime_factorization(2; k; h))
. |    {prime_factorization_existsLEMMA}
. |    Using:[n+1 | n+1 | g]
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. 4. ∃h:({2..(n+1)⁻}→ℕ).
.     Π↑{2..n+1⁻}(g) = Π↑{2..n+1⁻}(h)
.     & is_prime_factorization(2; (n+1); h)
. ⊢  ∃h:({2..(n+1)⁻}→ℕ).
.     n = Π↑{2..n+1⁻}(h) & is_prime_factorization(2; (n+1); h)
.   by SimilarTo: Hyp:-1
END
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