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prime_factorization_exists2  THM

⊢  ∀n:{1...}. ∃f:(Primeℕ→ℕ). f is a factorization of n
| by SimilarTo
|    Thm*  ∀n:{1...}.
|          ∃h:({2..(n+1)⁻}→ℕ).
|          n = Π↑{2..n+1⁻}(h) & is_prime_factorization(2; (n+1); h)
|    {prime_factorization_exists}
|
1. n : {1...}
2. ∃h:({2..(n+1)⁻}→ℕ).
|  n = Π↑{2..n+1⁻}(h) & is_prime_factorization(2; (n+1); h)
⊢  ∃f:(Primeℕ→ℕ). f is a factorization of n  by ExistHD Hyp:-1
|
2. h : {2..(n+1)⁻}→ℕ
3. n = Π↑{2..n+1⁻}(h)
4. is_prime_factorization(2; (n+1); h)
⊢  ∃f:(Primeℕ→ℕ). f is a factorization of n
| by Witness: complete_intseg_mset(2; (n+1); h)
|
⊢  complete_intseg_mset(2; (n+1); h) is a factorization of n
| by Analyze
|
|\
| 5. x : Primeℕ
| 6. n<x
| ⊢  complete_intseg_mset(2; (n+1); h)(x) = 0
|   by Rewrite by
|      Thm*  ∀a,b:ℤ, f:({a..b⁻}→ℕ), x:ℤ.
|            ¬a ≤ x < b ⇒ complete_intseg_mset(a; b; f)(x) = 0
|      {complete_intseg_mset_ismin}
|      THEN
|      Analyze
|
.\
. ⊢  n
. |  =
. |  Π↑{2..n+1⁻}(prime_mset_complete(complete_intseg_mset(2; (n+1); h)
. |                                 ))
. | by Π↑{2..n+1⁻}(h)
. |    =
. |    Π↑{2..n+1⁻}(prime_mset_complete(complete_intseg_mset(2;
. |                                                         (n+1);
. |                                                         h)))
. |    Asserted
. |    THENA
. |    (Analyze THEN FunExtensionality)
. |
. 5. x : {2..(n+1)⁻}
. ⊢  h(x) = prime_mset_complete(complete_intseg_mset(2; (n+1); h))(x)
. | by Decide: prime(x)
. |
. |\
. | 6. prime(x)
. | ⊢  h(x)
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. | |  =
. | |  prime_mset_complete(complete_intseg_mset(2; (n+1); h))(x)
. | | by Rewrite by
. | |    Thm*  ∀f:(Primeℕ→ℕ), x:Primeℕ.
. | |          prime_mset_complete(f)(x) = f(x)
. | |    {prime_mset_complete_isext}
. | |
. | ⊢  h(x) = complete_intseg_mset(2; (n+1); h)(x)
. |   by Rewrite by
. |      Thm*  ∀a,b:ℤ, f:({a..b⁻}→ℕ), x:{a..b⁻}.
. |            complete_intseg_mset(a; b; f)(x) = f(x)
. |      {complete_intseg_mset_isext}
. |
. .\
. . 6. ¬prime(x)
. . ⊢  h(x)
. . |  =
. . |  prime_mset_complete(complete_intseg_mset(2; (n+1); h))(x)
. . | by Rewrite by
. . |    Thm*  ∀f:(Primeℕ→ℕ), x:ℕ.
. . |          ¬prime(x) ⇒ prime_mset_complete(f)(x) = 0
. . |    {prime_mset_complete_ismin}
. . |
. . ⊢  h(x) = 0
. .   by {by contrapositive of Hyp:4 }
. .      ¬0<h(x)  Asserted THEN SimilarTo: Hyp:6 THEN BackThru: Hyp:4
END
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