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prime_factorization_existsLEMMA  THM

⊢  ∀k:{2...}, n:ℕ, g:({2..k⁻}→ℕ).
|  2 ≤ n < k+1
|  ⇒
|  (∀i:{2..k⁻}. n≤i ⇒ 0<g(i) ⇒ prime(i))
|  ⇒
|  (∃h:({2..k⁻}→ℕ).
|   Π↑{2..k⁻}(g) = Π↑{2..k⁻}(h) & is_prime_factorization(2; k; h))
| by (Analyze THEN (CompNatInd Concl)) ...ℕ
|
1. k : {2...}
2. n : ℕ
3. ∀n1:ℕ.
|  n1<n
|  ⇒
|  (∀g:({2..k⁻}→ℕ).
|   2 ≤ n1 < k+1
|   ⇒
|   (∀i:{2..k⁻}. n1≤i ⇒ 0<g(i) ⇒ prime(i))
|   ⇒
|   (∃h:({2..k⁻}→ℕ).
|    Π↑{2..k⁻}(g) = Π↑{2..k⁻}(h) & is_prime_factorization(2; k; h)))
4. g : {2..k⁻}→ℕ
5. 2≤n
6. n<k+1
7. ∀i:{2..k⁻}. n≤i ⇒ 0<g(i) ⇒ prime(i)
⊢  ∃h:({2..k⁻}→ℕ).
|  Π↑{2..k⁻}(g) = Π↑{2..k⁻}(h) & is_prime_factorization(2; k; h)
| by Decide: n = 2 ...wℕ
|
|\
| 8. n = 2
| ⊢  ∃h:({2..k⁻}→ℕ).
| |  Π↑{2..k⁻}(g) = Π↑{2..k⁻}(h) & is_prime_factorization(2; k; h)
| | by Witness: g ...ℕ
| |
| ⊢  is_prime_factorization(2; k; g)  by Analyze ...ℕ
| |
| 9. i : {2..k⁻}
| 10. 0<g(i)
| ⊢  prime(i)  by BackThru: Hyp:7 ...ℕ
|
.\
. 8. ¬n = 2
. ⊢  ∃h:({2..k⁻}→ℕ).
. |  Π↑{2..k⁻}(g) = Π↑{2..k⁻}(h) & is_prime_factorization(2; k; h)
. | by Decide: prime(n-1) ...wℕ
. |
. |\
. | 9. prime(n-1)
. | ⊢  ∃h:({2..k⁻}→ℕ).
. | |  Π↑{2..k⁻}(g) = Π↑{2..k⁻}(h) & is_prime_factorization(2; k; h)
. | | by Inst: Hyp:3 Using:[n-1 | g]
. | |
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. | .....antecedent .....
. | ⊢  ∀i:{2..k⁻}. n-1≤i ⇒ 0<g(i) ⇒ prime(i)
. |   by BackThru:
. |      Thm*  ∀a,b:ℤ, P:({a..b⁻}→Prop), n:{(a+1)..(b+1)⁻}.
. |            P(n-1)
. |            ⇒
. |            (∀i:{a..b⁻}. n≤i ⇒ P(i))
. |            ⇒
. |            (∀i:{a..b⁻}. n-1≤i ⇒ P(i))
. |      {extend_intseg_upperpart_down}
. |
. .\
. . 9. ¬prime(n-1)
. . ⊢  ∃h:({2..k⁻}→ℕ).
. . |  Π↑{2..k⁻}(g) = Π↑{2..k⁻}(h) & is_prime_factorization(2; k; h)
. . | by Inst:
. . |    Thm*  ∀k:{2...}, g:({2..k⁻}→ℕ), z:{2..k⁻}.
. . |          ¬prime(z)
. . |          ⇒
. . |          (∃g':({2..k⁻}→ℕ).
. . |           Π↑{2..k⁻}(g) = Π↑{2..k⁻}(g')
. . |           & g'(z) = 0
. . |           & (∀u:{2..k⁻}. z<u ⇒ g'(u) = g(u)))
. . |    {can_reduce_composite_factor2}
. . |    Using:[k | g | n-1] ...aℕ
. . |    THEN
. . |    ExistHD Hyp:-1
. . |
. . 10. g' : {2..k⁻}→ℕ
. . 11. Π↑{2..k⁻}(g) = Π↑{2..k⁻}(g')
. . 12. g'(n-1) = 0
. . 13. ∀u:{2..k⁻}. n-1<u ⇒ g'(u) = g(u)
. . ⊢  ∃h:({2..k⁻}→ℕ).
. . |  Π↑{2..k⁻}(g) = Π↑{2..k⁻}(h) & is_prime_factorization(2; k; h)
. . | by Inst: Hyp:3 Using:[n-1 | g'] ...ℕ
. . |
. . |\
. . | 14. i : {2..k⁻}
. . | 15. n-1≤i
. . | 16. 0<g'(i)
. . | ⊢  prime(i)  by BackThru: Hyp:7
. . | |
. . | ⊢  0<g(i)  by Rewrite by Hyp:13
. . |
. . .\
. . . 14. ∃h:({2..k⁻}→ℕ).
. . .     Π↑{2..k⁻}(g') = Π↑{2..k⁻}(h)

. . .     & is_prime_factorization(2; k; h)
. . . ⊢  ∃h:({2..k⁻}→ℕ).
. . .    Π↑{2..k⁻}(g) = Π↑{2..k⁻}(h) & is_prime_factorization(2; k; h)
. . .   by SimilarTo: Hyp:-1
END
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