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prime_factorization_includes_prime_divisors  THM

⊢  ∀a:ℕ, b:ℤ, f:({a..b⁻}→ℕ), p:ℕ.
|  is_prime_factorization(a; b; f)
|  ⇒
|  prime(p) ⇒ p | Π↑{a..b⁻}(f) ⇒ p ∈ {a..b⁻} & 0<f(p)
| by IntroduceGuardForms (<prop> & <prop>) 0
|    THEN
|    Def of Π↑{<int>..<int>⁻}(<exponents>) ...ℕ
|
1. a : ℕ
2. b : ℤ
3. f : {a..b⁻}→ℕ
4. p : ℕ
5. is_prime_factorization(a; b; f)
6. prime(p)
7. p | (Π i:{a..b⁻}. i↑f(i))
⊢  p ∈ {a..b⁻} & 0<f(p)
| by FwdThru:
|    Thm*  ∀p:ℕ.
|          prime(p)
|          ⇒
|          (∀a,b:ℤ, e:({a..b⁻}→ℕ).
|           a<b ⇒ p | (Π i:{a..b⁻}. e(i)) ⇒ (∃i:{a..b⁻}. p | e(i)))
|    {prime_divs_mul_via_intseg}
|    on [ Hyp:-1 ]
|
|\
| .....antecedent .....
| ⊢  a<b  by ¬b≤a  Asserted THEN Analyze
| |
| 8. b≤a
| ⊢  False
| | by Rewrite7 by
| |    Thm*  ∀f:(A→A→A), u:A, a,b:ℤ, e:({a..b⁻}→A).
| |          b≤a ⇒ (Iter(f;u) i:{a..b⁻}. e(i)) = u
| |    {iter_via_intseg_null}
| |
| 9. p | 1
| ⊢  False
|   by FwdThru:
|      Thm*  ∀b:ℤ. b | 1 ⇒ ¬prime(b)    {no_nat_prime_divs_one}
|      on [ p | 1 ]
|
.\
. 8. ∃i:{a..b⁻}. p | i↑f(i)
. ⊢  p ∈ {a..b⁻} & 0<f(p)  by Analyze-1
. |
. 8. i : {a..b⁻}
. 9. p | i↑f(i)
. ⊢  p ∈ {a..b⁻} & 0<f(p)
. | by FwdThru:
. |    Thm*  ∀p:ℕ. prime(p) ⇒ (∀b,z:ℤ. p | z↑b ⇒ b ≠ 0 & p | z)
. |    {prime_divs_exp}
. |    on [ Hyp:-1 ] ...ℕ
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. 10. f(i) ≠ 0
. 11. p | i
. ⊢  p ∈ {a..b⁻} & 0<f(p)
. | by Inst: is_prime_factorization(a; b; f) Using:[i]
. |
. 12. prime(i)
. ⊢  p ∈ {a..b⁻} & 0<f(p)
. | by FwdThru:
. |    Thm*  ∀x:{2...}, y:ℕ. prime(y) ⇒ x | y ⇒ x = y ∈ ℤ
. |    {prime_nat_divby_self_only}
. |    on [ p | i ] ...ℕ
. |
. |\
. | .....wf .....
. | ⊢  p ∈ {2...}
. |   by FwdThru:
. |      Thm*  ∀x:ℕ. prime(x) ⟺ 2≤x & ¬(∃i:{2..x⁻}. i | x)
. |      {prime_nat}
. |      on [ prime(p) ] ...ℕ
. |
. .\
. . 13. p = i
. . ⊢  p ∈ {a..b⁻} & 0<f(p)  by Analyze ...ℕ
. . |
. . 14. p ∈ {a..b⁻}
. . ⊢  0<f(p)  by Rewrite by p = i ∈ {a..b⁻} ...ℕ
END
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