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prime_factorization_limit  THM

⊢  ∀n:ℕ, f,g:(Primeℕ→ℕ).
|  f is a factorization of n
|  ⇒
|  g is a factorization of n
|  ⇒
|  (∀x:{2..(n+1)⁻}.
|   prime_mset_complete(f)(x) = prime_mset_complete(g)(x))
|  ⇒
|  f = g  by Auto THEN FunExtensionality
|
1. n : ℕ
2. f : Primeℕ→ℕ
3. g : Primeℕ→ℕ
4. f is a factorization of n
5. g is a factorization of n
6. ∀x:{2..(n+1)⁻}.
|  prime_mset_complete(f)(x) = prime_mset_complete(g)(x)
7. x : Primeℕ
⊢  f(x) = g(x)  by Analyze-1 THEN Decide: n<x
|
|\
| 7. x : ℕ
| 8. prime(x)
| 9. n<x
| ⊢  f(x) = g(x)
|   by FwdThru: Hyp:4 on [ n<x ] THEN FwdThru: Hyp:5 on [ n<x ]
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. 7. x : ℕ
. 8. prime(x)
. 9. ¬n<x
. ⊢  f(x) = g(x)  by Inst: Hyp:6 Using:[x]
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. |\
. | ⊢  2≤x
. |   by BackThru: Thm*  ∀x:ℕ. prime(x) ⇒ 2≤x    {natprimes_lb}
. |
. .\
. . 10. prime_mset_complete(f)(x) = prime_mset_complete(g)(x)
. . ⊢  f(x) = g(x)
. .   by Rewrite-1 by
. .      Thm*  ∀f:(Primeℕ→ℕ), x:Primeℕ.
. .             prime_mset_complete(f)(x) = f(x)
. .      {prime_mset_complete_isext}
END
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