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prime_factorization_mset_unique  THM

⊢  ∀n:ℕ, f,g:(Primeℕ→ℕ).
|  f is a factorization of n ⇒ g is a factorization of n ⇒ f = g
| by SimilarTo
|    Thm*  ∀n:ℕ, f,g:(Primeℕ→ℕ).
|          f is a factorization of n
|          ⇒
|          g is a factorization of n
|          ⇒
|          (∀x:{2..(n+1)⁻}.
|           prime_mset_complete(f)(x) = prime_mset_complete(g)(x))
|          ⇒
|          f = g
|    {prime_factorization_limit}
|    THEN
|    BackThru: Hyp:-1
|
1. n : ℕ
2. f : Primeℕ→ℕ
3. g : Primeℕ→ℕ
4. f is a factorization of n
5. g is a factorization of n
6. (∀x:{2..(n+1)⁻}.
|   prime_mset_complete(f)(x) = prime_mset_complete(g)(x))
|  ⇒
|  f = g
7. x : {2..(n+1)⁻}
⊢  prime_mset_complete(f)(x) = prime_mset_complete(g)(x)
| by Analyze5 THEN Analyze4
|
4. ∀x:Primeℕ. n<x ⇒ f(x) = 0
5. n = Π↑{2..n+1⁻}(prime_mset_complete(f))
6. ∀x:Primeℕ. n<x ⇒ g(x) = 0
7. n = Π↑{2..n+1⁻}(prime_mset_complete(g))
8. (∀x:{2..(n+1)⁻}.
|   prime_mset_complete(f)(x) = prime_mset_complete(g)(x))
|  ⇒
|  f = g
9. x : {2..(n+1)⁻}
⊢  prime_mset_complete(f)(x) = prime_mset_complete(g)(x)
| by ApFun: prime_mset_complete(f)
|           =
|           prime_mset_complete(g)
|           ∈ {2..(n+1)⁻}→ℕ
|       to: x
|
.....fun wf .....
⊢  prime_mset_complete(f) = prime_mset_complete(g) ∈ {2..(n+1)⁻}→ℕ
  by BackThru:
     Thm*  ∀a:{2...}, b:ℤ, g,h:({a..b⁻}→ℕ).
           is_prime_factorization(a; b; g)
           ⇒
           is_prime_factorization(a; b; h)
           ⇒
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      Π↑{a..b⁻}(g) = Π↑{a..b⁻}(h) ⇒ g = h
  {prime_factorization_unique}
  THEN
  BackThru:
  Thm*  ∀f:(Primeℕ→ℕ), b:ℕ.
      is_prime_factorization(2; b; prime_mset_complete(f))
  {prime_mset_c_is_prime_f}
END
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