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hanoi_seq_deepen_seq  THM

⊢  ∀a,z:ℤ, n:ℕ, s:({a...z}→{1...n}→Peg), n':ℕ.
|  n≤n'
|  ⇒
|  (∀h:({n+1...n'}→Peg).
|   s is a Hanoi(n disk) seq on a..z
|   ⇒
|   (s(?) {to n} ⊕ h {to n'}) is a Hanoi(n' disk) seq on a..z)
| by Auto
|
1. a : ℤ
2. z : ℤ
3. n : ℕ
4. s : {a...z}→{1...n}→Peg
5. n' : ℕ
6. n≤n'
7. h : {n+1...n'}→Peg
8. s is a Hanoi(n disk) seq on a..z
⊢  (s(?) {to n} ⊕ h {to n'}) is a Hanoi(n' disk) seq on a..z
| by SimilarTo Hyp:-1
|
8. x : {a...z}
9. x' : {a...z}
10. x+1 = x'
11. k : {1...n}
12. Moving disk k of n takes s(x) to s(x')
⊢  Moving disk k of n'
|  takes (s(?) {to n} ⊕ h {to n'})(x) to (s(?) {to n} ⊕ h {to n'})(x')
| by SimilarTo Hyp:-1 THENL [Analyze;SimilarTo Hyp:-1]
|
|\
| 12. ∀i:{1...n}. s(x,i) = s(x',i) ⟺ i ≠ k
| 13. i : {1...n'}
| ⊢  (s(?) {to n} ⊕ h {to n'})(x,i) = (s(?) {to n} ⊕ h {to n'})(x',i)
| |  ⟺
| |  i ≠ k  by Decide: i≤n
| |
| |\
| | 14. i≤n
| | ⊢  (s(?) {to n} ⊕ h {to n'})(x,i)
| | |  =
| | |  (s(?) {to n} ⊕ h {to n'})(x',i)
| | |  ⟺
| | |  i ≠ k  by Rewrite by Thm: hanoi_seq_deepen_loweq
| | |
| | ⊢  s(x,i) = s(x',i) ⟺ i ≠ k  by Rewrite by Hyp:12
| |
| .\
| . 14. ¬i≤n
| . ⊢  (s(?) {to n} ⊕ h {to n'})(x,i)
| .    =
| .    (s(?) {to n} ⊕ h {to n'})(x',i)
| .    ⟺
| .    i ≠ k  by Rewrite by Thm: hanoi_seq_deepen_higheq
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. 12. i : {1...k-1}
. 13. s(x,i) ≠ s(x,k) & s(x',i) ≠ s(x',k)
. ⊢  (s(?) {to n} ⊕ h {to n'})(x,i) ≠ (s(?) {to n} ⊕ h {to n'})(x,k)
.    & (s(?) {to n} ⊕ h {to n'})(x',i)
.      ≠
.      (s(?) {to n} ⊕ h {to n'})(x',k)
.   by Rewrite by Thm: hanoi_seq_deepen_loweq
END
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