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hanoi_sol2_ala_generalPROG_wf  THM

⊢  ∀n:ℕ, p,q:Peg.
|  p ≠ q
|  ⇒
|  (∀a:ℤ.
|   HanoiSTD(n disks; from: p; to: q; indexing from: a)
|   ∈ z:{a...}×({a...z}→{1...n}→Peg))
| by CompNatInd Concl THEN UnivCD
|
1. n : ℕ
2. ∀n1:ℕ.
|  n1<n
|  ⇒
|  (∀p,q:Peg.
|   p ≠ q
|   ⇒
|   (∀a:ℤ.
|    HanoiSTD(n1 disks; from: p; to: q; indexing from: a)
|    ∈ z:{a...}×({a...z}→{1...n1}→Peg)))
3. p : Peg
4. q : Peg
5. p ≠ q
6. a : ℤ
⊢  HanoiSTD(n disks; from: p; to: q; indexing from: a)
|  ∈ z:{a...}×({a...z}→{1...n}→Peg)
| by Compute
|      ⌈HanoiSTD(n disks; from: p; to: q; indexing from: a)⌉
|      *⇒
|      if n=₂0→ <a,λx,i. whatever>
|      else HanoiSTD(n-1 disks; from: p; to: otherPeg(p; q);
|                  indexing from: a)
|          /m,s1.
|          HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
|                  indexing from: m+1)
|          /z,s2.
|          <z
|          ,HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2> fi
|
⊢  if n=₂0→ <a,λx,i. whatever>
|  else HanoiSTD(n-1 disks; from: p; to: otherPeg(p; q);
|               indexing from: a)
|       /m,s1.
|       HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
|               indexing from: m+1)
|       /z,s2.
|       <z,HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2> fi
|  ∈ z:{a...}×({a...z}→{1...n}→Peg)  by SplitITE Concl
|
.....falsecase .....
7. n ≠ 0
⊢  (HanoiSTD(n-1 disks; from: p; to: otherPeg(p; q); indexing from: a)
|   /m,s1.
|   HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q; indexing from: m
|            +1)
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|   /z,s2. <z,HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2>)
|  ∈ z:{a...}×({a...z}→{1...n}→Peg)
| by Inst: Hyp:2 Using:[n-1 | p | otherPeg(p; q) | a]
|
|\
| .....antecedent .....
| ⊢  p ≠ otherPeg(p; q)
|   by BackThru:
|      Thm*  ∀x,y:Peg. x ≠ y ⇒ x ≠ otherPeg(x; y)
|      {hanoi_otherpeg_diff3}
|
.\
. 8. HanoiSTD(n-1 disks; from: p; to: otherPeg(p; q);
. |           indexing from: a)
. |  ∈ z:{a...}×({a...z}→{1...n-1}→Peg)
. ⊢  (HanoiSTD(n-1 disks; from: p; to: otherPeg(p; q);
. |            indexing from: a)
. |   /m,s1.
. |   HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
. |            indexing from: m+1)
. |   /z,s2. <z,HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2>)
. |  ∈ z:{a...}×({a...z}→{1...n}→Peg)
. | by Let
. |    (whatnot
. |     =
. |     HanoiSTD(n-1 disks; from: p; to: otherPeg(p; q);
. |              indexing from: a))
. |    THEN
. |    New:[m | s1] Analyze-2
. |    THEN
. |    Reduce Concl
. |
. 9. m : {a...}
. 10. s1 : {a...m}→{1...n-1}→Peg
. 11. <m,s1>
. |   =
. |   HanoiSTD(n-1 disks; from: p; to: otherPeg(p; q);
. |            indexing from: a)
. ⊢  (HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
. |            indexing from: m+1)
. |   /z,s2. <z,HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2>)
. |  ∈ z:{a...}×({a...z}→{1...n}→Peg)
. | by Inst: Hyp:2 Using:[n-1 | otherPeg(p; q) | q | m+1]
. |
. |\
. | .....antecedent .....
. | ⊢  otherPeg(p; q) ≠ q
. |   by BackThru:
. |      Thm*  ∀x,y:Peg. x ≠ y ⇒ otherPeg(x; y) ≠ y
. |      {hanoi_otherpeg_diff2}
. |
. .\
. . 12. HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
. . |            indexing from: m+1)
. . |   ∈ z:{m+1...}×({m+1...z}→{1...n-1}→Peg)
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. . ⊢  (HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
. . |            indexing from: m+1)
. . |   /z,s2.
. . |   <z,HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2>)
. . |  ∈ z:{a...}×({a...z}→{1...n}→Peg)
. . | by Let
. . |    (whatnot
. . |     =
. . |     HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
. . |              indexing from: m+1))
. . |    THEN
. . |    New:[z | s2] Analyze-2
. . |    THEN
. . |    Reduce Concl
. . |
. . 13. z : {m+1...}
. . 14. s2 : {m+1...z}→{1...n-1}→Peg
. . 15. <z,s2>
. .     =
. .     HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
. .              indexing from: m+1)
. . ⊢  <z,HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2>
. .    ∈ z:{a...}×({a...z}→{1...n}→Peg)  by Auto
END
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