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hanoi_sol2_ala_generalPROGworks  THM

⊢  ∀n:ℕ, p,q:Peg.
|  p ≠ q
|  ⇒
|  (∀a:ℤ.
|   HanoiSTD(n disks; from: p; to: q; indexing from: a)/z,s.
|   s is a Hanoi(n disk) seq on a..z
|   & s(a) = (λi.p) ∈ {1...n}→Peg
|   & s(z) = (λi.q) ∈ {1...n}→Peg)  by NatInd Concl
|
|\
| 1. p : Peg
| 2. q : Peg
| 3. p ≠ q
| 4. a : ℤ
| ⊢  HanoiSTD(0 disks; from: p; to: q; indexing from: a)/z,s.
| |  s is a Hanoi(0 disk) seq on a..z
| |  & s(a) = (λi.p) ∈ {1...0}→Peg
| |  & s(z) = (λi.q) ∈ {1...0}→Peg
| | by Compute
| |      ⌈HanoiSTD(0 disks; from: p; to: q; indexing from: a)/z,s.
| |       s is a Hanoi(0 disk) seq on a..z
| |       & s(a) = (λi.p) ∈ {1...0}→Peg
| |       & s(z) = (λi.q) ∈ {1...0}→Peg⌉
| |      *⇒
| |      (λx,i. whatever) is a Hanoi(0 disk) seq on a..a
| |      & (λx,i. whatever)(a) = (λi.p) ∈ {1...0}→Peg
| |      & (λx,i. whatever)(a) = (λi.q) ∈ {1...0}→Peg
| |
| |\
| | ⊢  (λx,i. whatever) is a Hanoi(0 disk) seq on a..a  by Analyze
| |
| |\
| | ⊢  (λx,i. whatever)(a) = (λi.p)  by FunExtensionality
| |
| .\
| . ⊢  (λx,i. whatever)(a) = (λi.q)  by FunExtensionality
|
.\
. 1. n : ℤ
. 2. 0<n
. 3. ∀p,q:Peg.
. |  p ≠ q
. |  ⇒
. |  (∀a:ℤ.
. |   HanoiSTD(n-1 disks; from: p; to: q; indexing from: a)/z,s.
. |   s is a Hanoi(n-1 disk) seq on a..z
. |   & s(a) = (λi.p) ∈ {1...n-1}→Peg
. |   & s(z) = (λi.q) ∈ {1...n-1}→Peg)
. 4. p : Peg
. 5. q : Peg
. 6. p ≠ q
. 7. a : ℤ
. ⊢  HanoiSTD(n disks; from: p; to: q; indexing from: a)/z,s.
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. |  s is a Hanoi(n disk) seq on a..z
. |  & s(a) = (λi.p) ∈ {1...n}→Peg
. |  & s(z) = (λi.q) ∈ {1...n}→Peg
. | by ¬p = otherPeg(p; q) | ¬otherPeg(p; q) = q  Asserted
. |
. |\
. | .....assertion .....
. | ⊢  ¬p = otherPeg(p; q)
. |   by BackThru:
. |      Thm*  ∀x,y:Peg. x ≠ y ⇒ x ≠ otherPeg(x; y)
. |      {hanoi_otherpeg_diff3}
. |
. |\
. | .....assertion .....
. | 8. ¬p = otherPeg(p; q)
. | ⊢  ¬otherPeg(p; q) = q
. |   by BackThru:
. |      Thm*  ∀x,y:Peg. x ≠ y ⇒ otherPeg(x; y) ≠ y
. |      {hanoi_otherpeg_diff2}
. |
. .\
. . 8. ¬p = otherPeg(p; q)
. . 9. ¬otherPeg(p; q) = q
. . ⊢  HanoiSTD(n disks; from: p; to: q; indexing from: a)/z,s.
. . |  s is a Hanoi(n disk) seq on a..z
. . |  & s(a) = (λi.p)
. . |  & s(z) = (λi.q)
. . | by Rewrite by Thm: hanoi_sol2_ala_generalPROGcomp
. . |
. . ⊢  (HanoiSTD(n-1 disks; from: p; to: otherPeg(p; q);
. . |            indexing from: a)
. . |   /m,s1.
. . |   HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
. . |            indexing from: m+1)
. . |   /z,s2.
. . |   <z,HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2>)
. . |  /z,s.
. . |  s is a Hanoi(n disk) seq on a..z
. . |  & s(a) = (λi.p)
. . |  & s(z) = (λi.q)
. . | by Let
. . |    (whatnot
. . |     =
. . |     HanoiSTD(n-1 disks; from: p; to: otherPeg(p; q);
. . |              indexing from: a))
. . |    THEN
. . |    New:[m | s1] Analyze-2

. . |    THEN
. . |    Inst: Hyp:3 Using:[p | otherPeg(p; q) | a]
. . |    THEN
. . |    Rewrite'-1 by Hyp:-2 THEN OnAllClauses Reduce
. . |
. . 10. m : {a...}
. . 11. s1 : {a...m}→{1...n-1}→Peg
. . 12. <m,s1>
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. . |   =
. . |   HanoiSTD(n-1 disks; from: p; to: otherPeg(p; q);
. . |            indexing from: a)
. . 13. s1 is a Hanoi(n-1 disk) seq on a..m
. . 14. s1(a) = (λi.p)
. . 15. s1(m) = (λi.otherPeg(p; q))
. . ⊢  (HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
. . |            indexing from: m+1)
. . |   /z,s2.
. . |   <z,HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2>)
. . |  /z,s.
. . |  s is a Hanoi(n disk) seq on a..z
. . |  & s(a) = (λi.p)
. . |  & s(z) = (λi.q)
. . | by Let
. . |    (whatnot
. . |     =
. . |     HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
. . |              indexing from: m+1))
. . |    THEN
. . |    New:[z | s2] Analyze-2
. . |    THEN
. . |    Inst: Hyp:3 Using:[otherPeg(p; q) | q | m+1]
. . |    THEN
. . |    Rewrite'-1 by Hyp:-2
. . |    THEN
. . |    OnAllClauses Reduce THEN Repeat Analyze-1
. . |
. . 16. z : {m+1...}
. . 17. s2 : {m+1...z}→{1...n-1}→Peg
. . 18. <z,s2>
. . |   =
. . |   HanoiSTD(n-1 disks; from: otherPeg(p; q); to: q;
. . |            indexing from: m+1)
. . 19. s2 is a Hanoi(n-1 disk) seq on m+1..z
. . 20. s2(m+1) = (λi.otherPeg(p; q))
. . 21. s2(z) = (λi.q)
. . ⊢  (HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2)
. . |  is a Hanoi(n disk) seq on a..z
. . |  & (HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2)(a)
. . |    =
. . |    (λi.p)
. . |  & (HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2. r1 @(m) r2)(z)
. . |    =
. . |    (λi.q)
. . | by Let (whatnot = HanoiHelper(n; s1; λi.p; s2; λi.q))
. . |    THEN
. . |    New:[r1 | r2] Analyze-2
. . |
. . 22. r1 : {a...m}→{1...n}→Peg
. . 23. r2 : {m+1...z}→{1...n}→Peg
. . 24. <r1,r2> = HanoiHelper(n; s1; λi.p; s2; λi.q)
. . ⊢  (<r1,r2>/r1,r2. r1 @(m) r2) is a Hanoi(n disk) seq on a..z
. . |  & (<r1,r2>/r1,r2. r1 @(m) r2)(a) = (λi.p)
. . |  & (<r1,r2>/r1,r2. r1 @(m) r2)(z) = (λi.q)
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. . | by <r1,r2>/r1,r2.
. . |    (r1 @(m) r2) is a Hanoi(n disk) seq on a..z
. . |    & (r1 @(m) r2)(a) = (λi.p)
. . |    & (r1 @(m) r2)(z) = (λi.q)
. . |    Asserted
. . |    THEN
. . |    (OnAllClauses Reduce)
. . |
. . .....assertion .....
. . ⊢  <r1,r2>/r1,r2.
. . |  (r1 @(m) r2) is a Hanoi(n disk) seq on a..z
. . |  & (r1 @(m) r2)(a) = (λi.p)
. . |  & (r1 @(m) r2)(z) = (λi.q)  by Subst: Hyp:24
. . |
. . ⊢  HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2.
. . |  (r1 @(m) r2) is a Hanoi(n disk) seq on a..z
. . |  & (r1 @(m) r2)(a) = (λi.p)
. . |  & (r1 @(m) r2)(z) = (λi.q)
. . | by (Inst:
. . |     Thm*  ∀n:ℕ⁺, a:ℤ, z:{a...}, m:{a...z-1}, f,g:({1...n}→Peg).
. . |            f(n) ≠ g(n)
. . |            ⇒
. . |            (∀s1:({a...m}→{1...n-1}→Peg),
. . |              s2:({m+1...z}→{1...n-1}→Peg).
. . |             s1 is a Hanoi(n-1 disk) seq on a..m
. . |             & s1(a) = f ∈ {1...n-1}→Peg
. . |             & s2 is a Hanoi(n-1 disk) seq on m+1..z
. . |             & s2(z) = g ∈ {1...n-1}→Peg
. . |             & s1(m) = s2(m+1)
. . |             & (∀i:{1...n-1}. s1(m,i) ≠ f(n) & s2(m+1,i) ≠ g(n))
. . |             ⇒
. . |             (HanoiHelper(n; s1; f; s2; g)/r1,r2.
. . |              (r1 @(m) r2) is a Hanoi(n disk) seq on a..z
. . |              & r1(a) = f
. . |              & r2(z) = g))
. . |     {hanoi_general_exists_lemma2PROGworks}
. . |     Using:[n | a | z | m | λi.p | λi.q | s1 | s2] ;

. . |     Guarding (<fun>(z)) Rewrite by Thm: hanoi_seq_join_part1)
. . |    THEN
. . |    Rewrite by Thm: hanoi_seq_join_part2
. . |
. . ⊢  HanoiHelper(n; s1; λi.p; s2; λi.q)/r1,r2.
. . |  (r1 @(m) r2) is a Hanoi(n disk) seq on a..z
. . |  & r1(a) = (λi.p)
. . |  & r2(z) = (λi.q)
. . | by BackThru:
. . |    Thm*  ∀n:ℕ⁺, a:ℤ, z:{a...}, m:{a...z-1}, f,g:({1...n}→Peg).
. . |          f(n) ≠ g(n)
. . |          ⇒
. . |          (∀s1:({a...m}→{1...n-1}→Peg),
. . |            s2:({m+1...z}→{1...n-1}→Peg).
. . |           s1 is a Hanoi(n-1 disk) seq on a..m
. . |           & s1(a) = f ∈ {1...n-1}→Peg
. . |           & s2 is a Hanoi(n-1 disk) seq on m+1..z
. . |           & s2(z) = g ∈ {1...n-1}→Peg
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. . |           & s1(m) = s2(m+1)
. . |           & (∀i:{1...n-1}. s1(m,i) ≠ f(n) & s2(m+1,i) ≠ g(n))
. . |           ⇒
. . |           (HanoiHelper(n; s1; f; s2; g)/r1,r2.
. . |            (r1 @(m) r2) is a Hanoi(n disk) seq on a..z
. . |            & r1(a) = f
. . |            & r2(z) = g))
. . |    {hanoi_general_exists_lemma2PROGworks}
. . |    THEN
. . |    Reduce Concl
. . |
. . |\
. . | 25. i : {1...n-1}
. . | ⊢  s1(m,i) ≠ p  by Rewrite by Hyp:15 THEN Reduce Concl
. . | |
. . | ⊢  otherPeg(p; q) ≠ p  by SimilarTo: Hyp:8
. . |
. . .\
. . . 25. i : {1...n-1}
. . . ⊢  s2(m+1,i) ≠ q  by Rewrite by Hyp:20 THEN Reduce Concl
END
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