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dec_pred_iff_some_boolfun  THM

⊢  (∀x:T. Dec(E(x))) ⟺ (∃f:(T→𝔹). ∀x:T. f(x) ⟺ E(x))  by GenUnivCD
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|\
| 1. T : Type
| 2. E : T→Prop
| 3. ∀x:T. Dec(E(x))
| ⊢  ∃f:(T→𝔹). ∀x:T. f(x) ⟺ E(x)
| | by DescribeFun: f by ∀x:T. ∃!b:𝔹. b ⟺ E(x) THEN Witness: f
| |
| 4. x : T
| ⊢  ∃!b:𝔹. b ⟺ E(x)
| | by FwdThru:
| |    Thm*  Dec(P) ⟺ (∃b:𝔹. b ⟺ P)    {decbl_iff_some_bool}
| |    on [ Dec(E(x)) ]
| |
| 5. ∃b:𝔹. b ⟺ E(x)
| ⊢  ∃!b:𝔹. b ⟺ E(x)  by SimilarTo Hyp:-1
| |
| 5. b : 𝔹
| 6. b ⟺ E(x)
| ⊢  b is the 𝔹 s.t. b ⟺ E(x)  by Analyze THEN CBV: b'
| |
| |\
| | ⊢  b ⟺ E(x)  by Hypothesis
| |
| .\
| . ⊢  ∀b':𝔹. (b' ⟺ E(x)) ⇒ b' = b  by Rewrite by b ⟺ E(x)
| . |
| . ⊢  ∀b':𝔹. (b' ⟺ b) ⇒ b' = b
| .   by 2 Times Analyze
| .      THEN
| .      BackThru:
| .      Thm*  ∀a,b:𝔹. (a ⟺ b) ⇒ a = b    {iff_imp_equal_bool}
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. 1. T : Type
. 2. E : T→Prop
. 3. ∃f:(T→𝔹). ∀x:T. f(x) ⟺ E(x)
. 4. x : T
. ⊢  Dec(E(x))  by Analyze-2 THEN Rewrite by Hyp:-2
END
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