```
dec_pred_imp_bool_decider_exists  THM

⊢  ∀A:Type, P:(A→Type).
|  (∀x:A. Dec(P(x))) ⇒ (∃!∈{p:(A→𝔹)| ∀x:A. p(x) ⟺ P(x) })
| by Guarding (∃∈<type>) UnivCD
|    THEN
|    FwdThru:
|    Thm*  (∀x:T. Dec(E(x))) ⟺ (∃f:(T→𝔹). ∀x:T. f(x) ⟺ E(x))
|    {dec_pred_iff_some_boolfun}
|    on [ ∀x:A. Dec(P(x)) ]
|
1. A : Type
2. P : A→Type
3. ∀x:A. Dec(P(x))
4. ∃f:(A→𝔹). ∀x:A. f(x) ⟺ P(x)
⊢  ∃!∈{p:(A→𝔹)| ∀x:A. p(x) ⟺ P(x) }
| by Analyze-1 THEN Witness: f THEN Rename-1: g THEN Analyze-1
|
4. f : A→𝔹
5. ∀x:A. f(x) ⟺ P(x)
6. g : A→𝔹
7. ∀x:A. g(x) ⟺ P(x)
⊢  f = g ∈ {p:(A→𝔹)| ∀x:A. p(x) ⟺ P(x) }  by Rewrite-3 by Hyp:-1
|
8. ∀x:A. f(x) ⟺ g(x)
⊢  f = g ∈ {p:(A→𝔹)| ∀x:A. p(x) ⟺ P(x) }  by Analyze
|
⊢  f = g  by FunExtensionality
|
9. x : A
⊢  f(x) = g(x)
  by BackThru: Thm*  ∀a,b:𝔹. (a ⟺ b) ⇒ a = b    {iff_imp_equal_bool}
     THEN
     Inst: Hyp:8 Using:[x]
END
```