```
equivrel_characterization  THM

⊢  ∀R:(A→A→Prop).
|  (EquivRel x,y:A. R(x,y))
|  ⟺
|  (∀x:A.
|   R(x,x) & (∀y:A. R(x,y) ⇒ R(y,x) & (∀z:A. R(y,z) ⇒ R(x,z))))
| by UnivCD THEN Analyze 0 THEN Analyze 0
|
|\
| 1. A : Type
| 2. R : A→A→Prop
| 3. EquivRel x,y:A. R(x,y)
| ⊢  ∀x:A.
| |  R(x,x) & (∀y:A. R(x,y) ⇒ R(y,x) & (∀z:A. R(y,z) ⇒ R(x,z)))
| | by Analyze 3 THEN Analyze 4
| |
| 3. Refl(A;x,y.R(x,y))
| 4. Sym x,y:A. R(x,y)
| 5. Trans x,y:A. R(x,y)
| ⊢  ∀x:A.
| |  R(x,x) & (∀y:A. R(x,y) ⇒ R(y,x) & (∀z:A. R(y,z) ⇒ R(x,z)))
| | by UnivCD
| |
| 6. x : A
| ⊢  R(x,x) & (∀y:A. R(x,y) ⇒ R(y,x) & (∀z:A. R(y,z) ⇒ R(x,z)))
| | by Analyze 0 THENL [Id;UnivCD]
| |
| |\
| | ⊢  R(x,x)  by UnfoldTopAb 3 THEN BackThru 3
| |
| .\
| . 7. y : A
| . 8. R(x,y)
| . ⊢  R(y,x) & (∀z:A. R(y,z) ⇒ R(x,z))
| . | by Analyze 0 THENL [Id;Auto]
| . |
| . |\
| . | ⊢  R(y,x)  by UnfoldTopAb 4 THEN BackThru 4
| . |
| . .\
| . . 9. z : A
| . . 10. R(y,z)
| . . ⊢  R(x,z)  by UnfoldTopAb 5 THEN FwdThru 5 [8;10]
|
.\
. 1. A : Type
. 2. R : A→A→Prop
. 3. ∀x:A.
. |  R(x,x) & (∀y:A. R(x,y) ⇒ R(y,x) & (∀z:A. R(y,z) ⇒ R(x,z)))
. ⊢  EquivRel x,y:A. R(x,y)  by Analyze 0 THENL [Id;Analyze 0]
. |
. |\
. | ⊢  Refl(A;x,y.R(x,y))  by Analyze 0
. | |
```

```
. | 4. a : A
. | ⊢  R(a,a)  by BackThru 3
. |
. |\
. | ⊢  Sym x,y:A. R(x,y)  by Analyze 0 THEN BackThru 3
. |
. .\
. . ⊢  Trans x,y:A. R(x,y)  by Analyze 0
. . |
. . 4. a : A
. . 5. b : A
. . 6. c : A
. . 7. R(a,b)
. . 8. R(b,c)
. . ⊢  R(a,c)  by Witness3: a
. . |
. . 9. R(a,a)
. . 10. ∀y:A. R(a,y) ⇒ R(y,a) & (∀z:A. R(y,z) ⇒ R(a,z))
. . ⊢  R(a,c)  by Witness-1: b
. . |
. . 11. R(a,b) ⇒ R(b,a) & (∀z:A. R(b,z) ⇒ R(a,z))
. . ⊢  R(a,c)  by BackThru -1
END
```