```
unique_fun_description  THM

⊢  ∀A:Type, B:(A→Type), P:(x:A→B(x)→Prop).
|  (∀x:A. ∃!y:B(x). P(x,y)) ⇒ (∃!f:(x:A→B(x)). ∀x:A. P(x,f(x)))
| by SimilarTo:
|    Thm*  (∀x:A. ∃!y:B(x). P(x,y))
|          ⇒
|          (∃f:(x:A→B(x)). ∀x:A. P(x,f(x)))
|    {fun_description}
|
1. A : Type
2. B : A→Type
3. P : x:A→B(x)→Prop
4. ∀x:A. ∃!y:B(x). P(x,y)
5. f : x:A→B(x)
6. ∀x:A. P(x,f(x))
⊢  f is the (x:A→B(x)) s.t. ∀x:A. P(x,f(x))
| by Analyze THEN Rename-2: g
|
7. g : x:A→B(x)
8. ∀x:A. P(x,g(x))
⊢  g = f  by FunExtensionality
|
9. x : A
⊢  g(x) = f(x)
| by Inst: Hyp:8 Using:[x] THEN Inst: Hyp:6 Using:[x]
|    THEN
|    Inst: Hyp:4 Using:[x]
|
10. P(x,g(x))
11. P(x,f(x))
12. ∃!y:B(x). P(x,y)
⊢  g(x) = f(x)  by Analyze-1 THEN Analyze-1 THEN CBV-1: z
|
12. y : B(x)
13. P(x,y)
14. ∀z:B(x). P(x,z) ⇒ z = y
⊢  g(x) = f(x)
| by Inst: Hyp:14 Using:[g(x)] THEN Inst: Hyp:14 Using:[f(x)]
|
15. g(x) = y
16. f(x) = y
⊢  g(x) = f(x)  by Trivial
END
```