```
unique_fun_description2  THM

⊢  ∀A:Type, B:(A→Type), P:(x:A→B(x)→Prop).
|  (∀x:A. ∃!y:B(x). P(x,y)) ⇒ (∃!∈{f:(x:A→B(x))| ∀x:A. P(x,f(x)) })
| by SimilarTo
|    Thm*  ∀B:(A→Type), P:(x:A→B(x)→Prop).
|          (∀x:A. ∃!y:B(x). P(x,y))
|          ⇒
|          (∃!f:(x:A→B(x)). ∀x:A. P(x,f(x)))
|    {unique_fun_description}
|
1. A : Type
2. B : A→Type
3. P : x:A→B(x)→Prop
4. ∀x:A. ∃!y:B(x). P(x,y)
5. ∃!f:(x:A→B(x)). ∀x:A. P(x,f(x))
⊢  ∃!∈{f:(x:A→B(x))| ∀x:A. P(x,f(x)) }  by Analyze-1
|
5. f : x:A→B(x)
6. f is the (x:A→B(x)) s.t. ∀x:A. P(x,f(x))
⊢  ∃!∈{f:(x:A→B(x))| ∀x:A. P(x,f(x)) }  by Analyze-1 THEN CBV-1: g
|
6. ∀x:A. P(x,f(x))
7. ∀g:(x:A→B(x)). (∀x:A. P(x,g(x))) ⇒ g = f
⊢  ∃!∈{f:(x:A→B(x))| ∀x:A. P(x,f(x)) }
| by Witness: f THEN Rename-1: g THEN Analyze-1
|
8. g : x:A→B(x)
9. ∀x:A. P(x,g(x))
⊢  f = g ∈ {f:(x:A→B(x))| ∀x:A. P(x,f(x)) }
  by Analyze THEN BackThru: Hyp:7
END
```