```
unique_indep_fun_description  THM

⊢  ∀A,B:Type{i}, P:(A→B→Prop{i}).
|  (∀x:A. ∃!y:B. P(x,y)) ⇒ (∃!f:(A→B). ∀x:A. P(x,f(x)))
| by SimilarTo:
|    Thm*  (∀x:A. ∃!y:B. P(x,y)) ⇒ (∃f:(A→B). ∀x:A. P(x,f(x)))
|    {indep_fun_description}
|
1. A : Type
2. B : Type
3. P : A→B→Prop
4. ∀x:A. ∃!y:B. P(x,y)
5. f : A→B
6. ∀x:A. P(x,f(x))
⊢  f is the (A→B) s.t. ∀x:A. P(x,f(x))  by Analyze THEN Rename-2: g
|
7. g : A→B
8. ∀x:A. P(x,g(x))
⊢  g = f  by FunExtensionality
|
9. x : A
⊢  g(x) = f(x)
| by Inst: Hyp:8 Using:[x] THEN Inst: Hyp:6 Using:[x]
|    THEN
|    Inst: Hyp:4 Using:[x]
|
10. P(x,g(x))
11. P(x,f(x))
12. ∃!y:B. P(x,y)
⊢  g(x) = f(x)  by Analyze-1 THEN Analyze-1 THEN CBV-1: z
|
12. y : B
13. P(x,y)
14. ∀z:B. P(x,z) ⇒ z = y
⊢  g(x) = f(x)
| by Inst: Hyp:14 Using:[g(x)] THEN Inst: Hyp:14 Using:[f(x)]
|
15. g(x) = y
16. f(x) = y
⊢  g(x) = f(x)  by Trivial
END
```