```
kleene_minimize_is_fp  THM

⊢  ∀f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }. f(mu(f))
| by ∀n:ℕ, f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }. mu(f) = n ⇒ f(mu(f))  Asserted
|
|\
| .....assertion .....
| ⊢  ∀n:ℕ, f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }. mu(f) = n ⇒ f(mu(f))
| | by CompNatInd Concl
| |
| 1. n : ℕ
| 2. ∀n1:ℕ.
| |  n1<n ⇒ (∀f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }. mu(f) = n1 ⇒ f(mu(f)))
| 3. f : {f:(ℕ→𝔹)| ∃x:ℕ. f(x) }
| 4. mu(f) = n
| ⊢  f(mu(f))
| | by Compute ⌈mu(f)⌉ *⇒ if f(0)→ 0 else 1+mu(λx.f(1+x)) fi
| |    THEN
| |    SplitOnConclITE
| |
| .....falsecase .....
| 5. ¬f(0)
| ⊢  f(1+mu(λx.f(1+x)))
| | by FwdThru:
| |    Thm* ∀f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }.
| |         ¬f(0) ⇒ (λx.f(1+x)) ∈ {f:(ℕ→𝔹)| ∃x:ℕ. f(x) }
| |    {kleene_tail}
| |    on [ ¬f(0) ]
| |    THEN
| |    (Inst: Hyp:2 Using:[mu(λx.f(1+x)) | λx.f(1+x)]
| |     THEN
| |     (Reduce Hyp:-1))
| |
| .....antecedent .....
| 6. (λx.f(1+x)) ∈ {f:(ℕ→𝔹)| ∃x:ℕ. f(x) }
| ⊢  mu(λx.f(1+x))<n
| | by Compute4 ⌈mu(f)⌉ *⇒ if f(0)→ 0 else 1+mu(λx.f(1+x)) fi
| |
| 4. if f(0)→ 0 else 1+mu(λx.f(1+x)) fi = n ∈ ℕ
| 5. ¬f(0)
| 6. (λx.f(1+x)) ∈ {f:(ℕ→𝔹)| ∃x:ℕ. f(x) }
| ⊢  mu(λx.f(1+x))<n
|   by Rewrite4 by
|      Thm* ∀b:𝔹, x,y:T. ¬b ⇒ if b→ x else y fi = y
|      {ite_rw_false}
|
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. 1. ∀n:ℕ, f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }. mu(f) = n ⇒ f(mu(f))
. ⊢  ∀f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }. f(mu(f))
.   by Auto THEN BackThru: Hyp:1 Using:[mu(f)]
END
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