```
kleene_minimize_is_lb  THM

⊢  ∀f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }, i:ℕ. i<mu(f) ⇒ ¬f(i)
| by ∀n:ℕ, f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }.
|    mu(f) = n ⇒ (∀i:ℕ. i<mu(f) ⇒ ¬f(i))
|    Asserted
|
|\
| .....assertion .....
| ⊢  ∀n:ℕ, f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }.
| |  mu(f) = n ⇒ (∀i:ℕ. i<mu(f) ⇒ ¬f(i))  by CompNatInd Concl
| |
| 1. n : ℕ
| 2. ∀n1:ℕ.
| |  n1<n
| |  ⇒
| |  (∀f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }.
| |   mu(f) = n1 ⇒ (∀i:ℕ. i<mu(f) ⇒ ¬f(i)))
| ⊢  ∀f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }. mu(f) = n ⇒ (∀i:ℕ. i<mu(f) ⇒ ¬f(i))
| | by Compute ⌈mu(f)⌉ *⇒ if f(0)→ 0 else 1+mu(λx.f(1+x)) fi
| |
| ⊢  ∀f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }.
| |  if f(0)→ 0 else 1+mu(λx.f(1+x)) fi = n ∈ ℕ
| |  ⇒
| |  (∀i:ℕ. i<if f(0)→ 0 else 1+mu(λx.f(1+x)) fi ⇒ ¬f(i))
| | by Analyze
| |
| 3. f : {f:(ℕ→𝔹)| ∃x:ℕ. f(x) }
| ⊢  if f(0)→ 0 else 1+mu(λx.f(1+x)) fi = n ∈ ℕ
| |  ⇒
| |  (∀i:ℕ. i<if f(0)→ 0 else 1+mu(λx.f(1+x)) fi ⇒ ¬f(i))
| | by SplitOnConclITE
| |    THEN
| |    Try
| |    FwdThru:
| |    Thm* ∀f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }.
| |         ¬f(0) ⇒ (λx.f(1+x)) ∈ {f:(ℕ→𝔹)| ∃x:ℕ. f(x) }
| |    {kleene_tail}
| |    on [ ¬f(0) ]
| |
| 4. ¬f(0)
| 5. 1+mu(λx.f(1+x)) = n ∈ ℕ
| 6. i : ℕ
| 7. i<1+mu(λx.f(1+x))
| 8. (λx.f(1+x)) ∈ {f:(ℕ→𝔹)| ∃x:ℕ. f(x) }
| ⊢  ¬f(i)  by Decide: i = 0
| |
| |\
| | 9. i = 0
| | ⊢  ¬f(i)  by Rewrite by i = 0 ∈ ℕ
| |
| .\
| . 9. ¬i = 0
| . ⊢  ¬f(i)  by Inst: Hyp:2 Using:[mu(λx.f(1+x)) | λx.f(1+x) | i-1]
| . |
```

```
| . 10. ¬(λx.f(1+x))(i-1)
| . ⊢  ¬f(i)  by Reduce Hyp:-1 THEN ArithSimp Hyp:-1
|
.\
. 1. ∀n:ℕ, f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }.
.    mu(f) = n ⇒ (∀i:ℕ. i<mu(f) ⇒ ¬f(i))
. ⊢  ∀f:{f:(ℕ→𝔹)| ∃x:ℕ. f(x) }, i:ℕ. i<mu(f) ⇒ ¬f(i)
.   by Auto THEN BackThru: Hyp:1 Using:[mu(f)]
END
```