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kleene_minimize_wf  THM

⊢  mu ∈ {f:(ℕ→𝔹)| ∃x:ℕ. f(x) }→ℕ
| by ∀n:ℕ, f:(ℕ→𝔹). f(n) ⇒ mu(f) ∈ ℕ  Asserted
|
|\
| .....assertion .....
| ⊢  ∀n:ℕ, f:(ℕ→𝔹). f(n) ⇒ mu(f) ∈ ℕ
| | by CompNatInd Concl THEN UnivCD
| |
| 1. n : ℕ
| 2. ∀n1:ℕ. n1<n ⇒ (∀f:(ℕ→𝔹). f(n1) ⇒ mu(f) ∈ ℕ)
| 3. f : ℕ→𝔹
| 4. f(n)
| ⊢  mu(f) ∈ ℕ
| | by Compute ⌈mu(f)⌉ *⇒ if f(0)→ 0 else 1+mu(λx.f(1+x)) fi
| |
| ⊢  if f(0)→ 0 else 1+mu(λx.f(1+x)) fi ∈ ℕ  by SplitOnConclITE
| |
| .....falsecase .....
| 5. ¬f(0)
| ⊢  1+mu(λx.f(1+x)) ∈ ℕ
| | by BackThru: Hyp:2 Using:[n-1] THEN Reduce Concl ;
| |    BackThru: Thm* ∀i,j:ℕ. i+j ∈ ℕ    {add_nat_wf}
| |
| |\
| | .....wf .....
| | ⊢  1 ∈ ℕ  by Auto
| |
| .\
| . .....wf .....
| . ⊢  mu(λx.f(1+x)) ∈ ℕ
| . | by BackThru: Hyp:2 Using:[n-1] THEN Reduce Concl
| . |
| . |\
| . | .....wf .....
| . | ⊢  n-1 ∈ ℕ  by n ≠ 0 ∈ ℕ  Asserted
| . | |
| . | .....assertion .....
| . | ⊢  n ≠ 0 ∈ ℕ  by SimilarTo: Hyp:5
| . | |
| . | 5. n = 0 ∈ ℕ
| . | ⊢  f(0)  by Rewrite by n = 0 ∈ ℕ
| . |
| . .\
| . . ⊢  f(1+n-1)  by ArithSimp 0
|
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. 1. ∀n:ℕ, f:(ℕ→𝔹). f(n) ⇒ mu(f) ∈ ℕ
. ⊢  mu ∈ {f:(ℕ→𝔹)| ∃x:ℕ. f(x) }→ℕ
. | by With (Void→Void) (New:f FunExtensionality)
. |
. |\
. | .....wf .....
. | ⊢  mu ∈ Void→Void  by Def
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. .\
. . 2. f : {f:(ℕ→𝔹)| ∃x:ℕ. f(x) }
. . ⊢  mu(f) ∈ ℕ  by 2 Times Analyze-1
. . |
. . 2. f : ℕ→𝔹
. . 3. x : ℕ
. . 4. f(x)
. . ⊢  mu(f) ∈ ℕ  by BackThru: Hyp:1 Using:[x]
END
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