```
sfa_doc_factorial2_wf  THM

⊢  ∀x:{x:ℕ| (x rem 2) = 0 }. x!₂ ∈ ℕ
| by ∀x:ℕ. x!₂ ∈ ℕ(given (x rem 2) = 0)  Asserted
|
|\
| .....assertion .....
| ⊢  ∀x:ℕ. x!₂ ∈ ℕ(given (x rem 2) = 0)  by CompNatInd Concl
| |
| 1. x : ℕ
| 2. ∀x1:ℕ. x1<x ⇒ x1!₂ ∈ ℕ(given (x1 rem 2) = 0)
| ⊢  x!₂ ∈ ℕ(given (x rem 2) = 0)
| | by Compute ⌈x!₂⌉ *⇒ if x=₂0→ 1 else x•(x-2)!₂ fi
| |    THEN
| |    SplitITE Concl
| |
| |\
| | .....truecase .....
| | 3. x = 0
| | ⊢  1 ∈ ℕ(given (x rem 2) = 0)  by Auto
| |
| .\
| . .....falsecase .....
| . 3. x ≠ 0
| . ⊢  x•(x-2)!₂ ∈ ℕ(given (x rem 2) = 0)  by Decide: (x rem 2) = 0
| . |
| . |\
| . | 4. (x rem 2) = 0
| . | ⊢  x•(x-2)!₂ ∈ ℕ(given (x rem 2) = 0)  by Analyze
| . | |
| . | 5. (x rem 2) = 0
| . | ⊢  x•(x-2)!₂ ∈ ℕ  by 2≤x  Asserted
| . | |
| . | |\
| . | | .....assertion .....
| . | | ⊢  2≤x  by ¬x = 1  Asserted
| . | | |
| . | | .....assertion .....
| . | | ⊢  ¬x = 1  by Analyze
| . | | |
| . | | 6. x = 1
| . | | ⊢  False  by Rewrite4 by x = 1
| . | | |
| . | | 7. (1 rem 2) = 0
| . | | ⊢  False  by Compute-1 (⌈1 rem 2⌉ *⇒ 1) = 0
| . | |
| . | .\
| . | . 6. 2≤x
| . | . ⊢  x•(x-2)!₂ ∈ ℕ
| . | . | by BackThru: Thm* ∀i,j:ℕ. i•j ∈ ℕ    {multiply_nat_wf}
| . | . |
| . | . |\
| . | . | .....wf .....
| . | . | ⊢  x ∈ ℕ  by Trivial
| . | . |
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| . | . .\
| . | . . .....wf .....
| . | . . ⊢  (x-2)!₂ ∈ ℕ  by FwdThru: Hyp:2 on [ x-2<x ]
| . | . . |
| . | . . |\
| . | . . | .....wf .....
| . | . . | ⊢  x-2 ∈ ℕ  by ¬x = 1  Asserted THEN Analyze
| . | . . |
| . | . . .\
| . | . . . 7. (x-2)!₂ ∈ ℕ(given ((x-2) rem 2) = 0)
| . | . . . ⊢  (x-2)!₂ ∈ ℕ  by ((x-2) rem 2) = 0  Asserted
| . | . . . |
| . | . . . |\
| . | . . . | .....assertion .....
| . | . . . | ⊢  ((x-2) rem 2) = 0
| . | . . . |   by Rewrite← by
| . | . . . |      Thm* ∀a:ℕ, n:ℕ⁺. a≥n ⇒ (a rem n) = ((a-n) rem n)
| . | . . . |      {rem_rec_case}
| . | . . . |
| . | . . . .\
| . | . . . . 8. ((x-2) rem 2) = 0
| . | . . . . ⊢  (x-2)!₂ ∈ ℕ  by Rename-1: w THEN Witness7: w
| . |
| . .\
| . . 4. ¬(x rem 2) = 0
| . . ⊢  x•(x-2)!₂ ∈ ℕ(given (x rem 2) = 0)  by Auto
|
.\
. 1. ∀x:ℕ. x!₂ ∈ ℕ(given (x rem 2) = 0)
. ⊢  ∀x:{x:ℕ| (x rem 2) = 0 }. x!₂ ∈ ℕ  by SimilarTo: Hyp:-1
. |
. 1. x : {x:ℕ| (x rem 2) = 0 }
. 2. x!₂ ∈ ℕ(given (x rem 2) = 0)
. ⊢  x!₂ ∈ ℕ  by Analyze-2
. |
. 1. x : ℕ
. 2. (x rem 2) = 0
. 3. x!₂ ∈ ℕ(given (x rem 2) = 0)
. ⊢  x!₂ ∈ ℕ  by Rename-2: w THEN Witness-1: w
END
```