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sfa_doc_find_intlist_great_bound  THM

⊢  x:(ℤ List)→{y:ℤ| y greater-bounds x }  by Auto
|
1. x : ℤ List
⊢  {y:ℤ| y greater-bounds x }  by Def of y greater-bounds x
|
⊢  {y:ℤ| ∀i:ℕ||x||. x[i]<y }  by ListInd Hyp:1
|
|\
| ⊢  {y:ℤ| ∀i:ℕ||nil||. nil[i]<y }  by Reduce Concl
| |
| ⊢  {y:ℤ| ∀i:ℕ0. nil[i]<y }  by Witness: -3
|
.\
. 2. u : ℤ
. 3. v : ℤ List
. 4. {y:ℤ| ∀i:ℕ||v||. v[i]<y }
. ⊢  {y:ℤ| ∀i:ℕ||u.v||. (u.v)[i]<y }  by New:w Analyze-1
. |
. 4. w : ℤ
. 5. ∀i:ℕ||v||. v[i]<w  [not for witness]
. ⊢  {y:ℤ| ∀i:ℕ||u.v||. (u.v)[i]<y }  by Decide: u<w
. |
. |\
. | 6. u<w
. | ⊢  {y:ℤ| ∀i:ℕ||u.v||. (u.v)[i]<y }  by Witness: w
. | |
. | 5. ∀i:ℕ||v||. v[i]<w
. | 6. u<w
. | 7. i : ℕ||u.v||
. | ⊢  (u.v)[i]<w  by SideProof 1
. |
. .\
. . 6. ¬u<w
. . ⊢  {y:ℤ| ∀i:ℕ||u.v||. (u.v)[i]<y }  by Witness: u+2
. . |
. . 5. ∀i:ℕ||v||. v[i]<w
. . 6. ¬u<w
. . 7. i : ℕ||u.v||
. . ⊢  (u.v)[i]<u+2  by SideProof 2
END
```

```
sfa_doc_find_intlist_great_bound Side Proof 1  THM

1. ℤ List
2. u : ℤ
3. v : ℤ List
4. w : ℤ
5. ∀i:ℕ||v||. v[i]<w
6. u<w
7. i : ℕ||u.v||
⊢  (u.v)[i]<w  by Decide: 0<i
|
|\
| 8. 0<i
| ⊢  (u.v)[i]<w
| | by Rewrite by
| |    Thm* ∀a:T, as:T List, i:ℤ.
| |         0<i ⇒ i≤||as|| ⇒ (a.as)[i] = as[(i-1)]
| |    {select_cons_tl}
| |
| ⊢  v[(i-1)]<w  by BackThru: Hyp:5
|
.\
. 8. ¬0<i
. ⊢  (u.v)[i]<w
.   by Rewrite by
.      Thm* ∀a:T, as:T List, i:ℤ. i≤0 ⇒ (a.as)[i] = a
.      {select_cons_hd}
END
```

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sfa_doc_find_intlist_great_bound Side Proof 2  THM

1. ℤ List
2. u : ℤ
3. v : ℤ List
4. w : ℤ
5. ∀i:ℕ||v||. v[i]<w
6. ¬u<w
7. i : ℕ||u.v||
⊢  (u.v)[i]<u+2  by Decide: 0<i
|
|\
| 8. 0<i
| ⊢  (u.v)[i]<u+2
| | by Rewrite by
| |    Thm* ∀a:T, as:T List, i:ℤ.
| |         0<i ⇒ i≤||as|| ⇒ (a.as)[i] = as[(i-1)]
| |    {select_cons_tl}
| |
| ⊢  v[(i-1)]<u+2  by Inst: Hyp:5 Using:[i-1]
|
.\
. 8. ¬0<i
. ⊢  (u.v)[i]<u+2
.   by Rewrite by
.      Thm* ∀a:T, as:T List, i:ℤ. i≤0 ⇒ (a.as)[i] = a
.      {select_cons_hd}
END
```