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sfa_doc_find_small_greater_bound  THM

⊢  ∩k:ℕ. x:(ℕk List)→{y:ℕ(k+1)| y greater-bounds x }  by Analyze
|
1. k : ℕ  [not for witness]
2. x : ℕk List
⊢  {y:ℕ(k+1)| y greater-bounds x }  by ListInd Hyp:-1
|
|\
| ⊢  {y:ℕ(k+1)| y greater-bounds nil }
|   by Def of <int> greater-bounds <int list> THEN Reduce Concl
|      THEN
|      Witness: 0
|
.\
. 3. u : ℕk
. 4. v : ℕk List
. 5. {y:ℕ(k+1)| y greater-bounds v }
. ⊢  {y:ℕ(k+1)| y greater-bounds u.v }  by Analyze-1
. |
. 5. y : ℕ(k+1)
. 6. y greater-bounds v  [not for witness]
. ⊢  {y:ℕ(k+1)| y greater-bounds u.v }  by Decide: u<y
. |
. |\
. | 7. u<y
. | ⊢  {y:ℕ(k+1)| y greater-bounds u.v }  by Witness: y
. | |
. | 1. k : ℕ
. | 2. ℕk List
. | 3. u : ℕk
. | 4. v : ℕk List
. | 5. y : ℕ(k+1)
. | 6. y greater-bounds v
. | 7. u<y
. | ⊢  y greater-bounds u.v
. | | by Def of <int> greater-bounds <int list> THEN Decide: 0<i
. | |
. | |\
. | | 8. i : ℕ||u.v||
. | | 9. 0<i
. | | ⊢  (u.v)[i]<y
. | | | by Rewrite by
. | | |    Thm* ∀a:T, as:T List, i:ℤ.
. | | |         0<i ⇒ i≤||as|| ⇒ (a.as)[i] = as[(i-1)]
. | | |    {select_cons_tl}
. | | |
. | | ⊢  v[(i-1)]<y  by BackThru: Hyp:6
. | | |
. | | ⊢  i-1 ∈ ℕ||v||  by Analyze8
. | |
. | .\
. | . 8. i : ℕ||u.v||
. | . 9. ¬0<i
. | . ⊢  (u.v)[i]<y
```

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. | .   by Rewrite by
. | .      Thm* ∀a:T, as:T List, i:ℤ. i≤0 ⇒ (a.as)[i] = a
. | .      {select_cons_hd}
. |
. .\
. . 7. ¬u<y
. . ⊢  {y:ℕ(k+1)| y greater-bounds u.v }  by Witness: u+1
. . |
. . 1. k : ℕ
. . 2. ℕk List
. . 3. u : ℕk
. . 4. v : ℕk List
. . 5. y : ℕ(k+1)
. . 6. y greater-bounds v
. . 7. ¬u<y
. . ⊢  u+1 greater-bounds u.v  by Reduce Hyp:-1
. . |
. . ⊢  u+1 greater-bounds u.v
. . | by Def of <int> greater-bounds <int list>
. . |
. . 8. i : ℕ||u.v||
. . ⊢  (u.v)[i]<u+1  by Reduce Hyp:8 THEN Decide: 0<i
. . |
. . |\
. . | 8. i : ℕ(||v||+1)
. . | 9. 0<i
. . | ⊢  (u.v)[i]<u+1
. . | | by Rewrite by
. . | |    Thm* ∀a:T, as:T List, i:ℤ.
. . | |         0<i ⇒ i≤||as|| ⇒ (a.as)[i] = as[(i-1)]
. . | |    {select_cons_tl}
. . | |
. . | ⊢  v[(i-1)]<u+1  by Inst: Hyp:6 Using:[i-1]
. . |
. . .\
. . . 8. i : ℕ(||v||+1)
. . . 9. ¬0<i
. . . ⊢  (u.v)[i]<u+1
. . .   by Rewrite by
. . .      Thm* ∀a:T, as:T List, i:ℤ. i≤0 ⇒ (a.as)[i] = a
. . .      {select_cons_hd}
END
```