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sfa_doc_listind_via_so  THM

⊢  ∀A:Type, Q:Prop, R:(A→(A List)→Prop→Prop).
|  ∃P:((A List)→Prop).
|  (P(nil) ⟺ Q) & (∀a:A, x:A List. P(a.x) ⟺ R(a,x,P(x)))
| by RemoveGuards 0
|
1. A : Type
2. Q : Prop
3. R : A→(A List)→Prop→Prop
⊢  ∃P:((A List)→Prop).
|  (P(nil) ⟺ Q) & (∀a:A, x:A List. P(a.x) ⟺ R(a,x,P(x)))
| by Witness:
|      λx.Case of x; nil → Q ; a.x, rec:r → R(a,x,r)
|      with type (A List)→Prop
|    THEN
|    ReduceSOAps Concl
|
|\
| 4. x : A List
| ⊢  (Case of x; nil → Q ; a.x, rec:r → R(a,x,r)) ∈ Prop
|   by ListInd Hyp:-1 THEN Reduce Concl
|
.\
. 4. (λx.Case of x; nil → Q ; a.x, rec:r → R(a,x,r)) ∈ (A List)→Prop
. ⊢  ((Case of nil; nil → Q ; a.x, rec:r → R(a,x,r)) ⟺ Q)
. |  & (∀a:A, x:A List.
. |     (Case of a.x; nil → Q ; a.x, rec:r → R(a,x,r))
. |     ⟺
. |     R(a,x,Case of x; nil → Q ; a.x, rec:r → R(a,x,r)))
. | by Reduce Concl
. |
. ⊢  (Q ⟺ Q)
. |  & (∀a:A, x:A List.
. |     R(a,x,Case of x; nil → Q ; a.x, rec:r → R(a,x,r))
. |     ⟺
. |     R(a,x,Case of x; nil → Q ; a.x, rec:r → R(a,x,r)))
. | by Guarding (R(a,x,<arg>) ⟺ R(a,x,<arg>)) Auto
. |
. 5. a : A
. 6. x : A List
. ⊢  R(a,x,Case of x; nil → Q ; a.x, rec:r → R(a,x,r))
.    ⟺
.    R(a,x,Case of x; nil → Q ; a.x, rec:r → R(a,x,r))
.   by ApFun: Hyp:4 to: x
END
```