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sfa_doc_nat_sqrt  THM

⊢  ∀x:ℕ. ∃y:ℕ. y·y≤x & x<(y+1)·(y+1)  by NatInd Concl
|
|\
| .....basecase .....
| ⊢  ∃y:ℕ. y·y≤0 & 0<(y+1)·(y+1)  by Witness: 0
|
.\
. .....upcase .....
. 1. x : ℤ
. 2. 0<x
. 3. ∃y:ℕ. y·y≤x-1 & x-1<(y+1)·(y+1)
. ⊢  ∃y:ℕ. y·y≤x & x<(y+1)·(y+1)  by ExistHD Hyp:-1
. |
. 3. y : ℕ
. 4. y·y≤x-1
. 5. x-1<(y+1)·(y+1)
. ⊢  ∃y:ℕ. y·y≤x & x<(y+1)·(y+1)  by Decide: x<(y+1)·(y+1)
. |
. |\
. | 6. x<(y+1)·(y+1)
. | ⊢  ∃y:ℕ. y·y≤x & x<(y+1)·(y+1)  by Witness: y
. |
. .\
. . 6. ¬x<(y+1)·(y+1)
. . ⊢  ∃y:ℕ. y·y≤x & x<(y+1)·(y+1)  by Witness: y+1
END
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