```
sfa_doc_ntuple_contains_wf  THM

⊢  ∀A:Type, P:(A→Prop), n:ℕ, X:(A^n). (∃ u in X: A^n. P(u)) ∈ Prop
| by Guarding (∀n:ℕ. <prop>) Auto THEN CompNatInd Concl THEN UnivCD
|
1. A : Type
2. P : A→Prop
3. n : ℕ
4. ∀n1:ℕ. n1<n ⇒ (∀X:(A^n1). (∃ u in X: A^n1. P(u)) ∈ Prop)
5. X : A^n
⊢  (∃ u in X: A^n. P(u)) ∈ Prop
| by Compute
|      (⌈∃ u in X: A^n. P(u)⌉
|       *⇒
|       n = 1 ∈ ℕ & P(X)
|       ∨ n≥2 & (X/a,rest. P(a) ∨ (∃ u in rest: A^(n-1). P(u))))
|      ∈ Prop
|
⊢  (n = 1 ∈ ℕ & P(X)
|   ∨ n≥2 & (X/a,rest. P(a) ∨ (∃ u in rest: A^(n-1). P(u))))
|  ∈ Prop  by AndToCandLocal (2 Times Analyze)
|
|\
| 6. n = 1 ∈ ℕ
| ⊢  X ∈ A  by ChangeToEqType: A^n
| |
| .....equality .....
| ⊢  A = (A^n)  by Subst: n = 1 ∈ ℕ
| |
| ⊢  A = (A^1)  by Compute ⌈A^1⌉ *⇒ A
|
.\
. 6. n≥2
. ⊢  (X/a,rest. P(a) ∨ (∃ u in rest: A^(n-1). P(u))) ∈ Prop
. | by ChangeToEqType5: A×(A^(n-1))
. |
. |\
. | 5. X : A×(A^(n-1))
. | 6. n≥2
. | ⊢  (X/a,rest. P(a) ∨ (∃ u in rest: A^(n-1). P(u))) ∈ Prop
. | | by Analyze-2 THEN Reduce Concl
. | |
. | 5. X1 : A
. | 6. X2 : A^(n-1)
. | 7. n≥2
. | ⊢  (P(X1) ∨ (∃ u in X2: A^(n-1). P(u))) ∈ Prop  by Analyze
. | |
. | |\
. | | ⊢  P(X1) ∈ Prop  by Auto
. | |
. | .\
. | . ⊢  (∃ u in X2: A^(n-1). P(u)) ∈ Prop  by BackThru: Hyp:4
. |
. .\
. . .....equality .....
```

```
. . ⊢  (A^n) = A×(A^(n-1))
. .   by BackThru:
. .      Thm* ∀n:ℕ. n≥2 ⇒ (A^n) = A×(A^(n-1))
. .      {sfa_doc_ntuple_is_prod}
END
```