```
sfa_doc_ycomb_factorial_typing  THM

⊢  Y(λf,x. if x=₂0→ 1 else x·f(x-1) fi) ∈ ℕ→ℕ  by FunExtensionality
|
|\
| .....wf .....
| ⊢  Y(λf,x. if x=₂0→ 1 else x·f(x-1) fi) ∈ Void→Void
|   by Compute
|        ⌈Y(λf,x. if x=₂0→ 1 else x·f(x-1) fi)⌉
|        *⇒
|        λx.if x=₂0→ 1
|           else x·(λx.(λf,x. if x=₂0→ 1 else x·f(x-1) fi)(x(x)))
|                  ((λx.(λf,x. if x=₂0→ 1 else x·f(x-1) fi)(x(x)))
|                  ,x-1) fi
|
.\
. 1. x : ℕ
. ⊢  Y((λf,x. if x=₂0→ 1 else x·f(x-1) fi),x) ∈ ℕ
. | by CompNatInd Hyp:-1
. |
. 2. ∀x1:ℕ. x1<x ⇒ Y((λf,x. if x=₂0→ 1 else x·f(x-1) fi),x1) ∈ ℕ
. ⊢  Y((λf,x. if x=₂0→ 1 else x·f(x-1) fi),x) ∈ ℕ
. | by Compute
. |      Y((λf,x. if x=₂0→ 1 else x·f(x-1) fi),x)
. |      *⇒
. |      if x=₂0→ 1
. |      else x·Y((λf,x. if x=₂0→ 1 else x·f(x-1) fi),x-1) fi
. |
. ⊢  if x=₂0→ 1 else x·Y((λf,x. if x=₂0→ 1 else x·f(x-1) fi),x-1) fi
. |  ∈ ℕ  by SplitOnConclITE
. |
. .....falsecase .....
. 3. ¬x = 0
. ⊢  x·Y((λf,x. if x=₂0→ 1 else x·f(x-1) fi),x-1) ∈ ℕ
. | by BackThru: Thm* ∀i,j:ℕ. i·j ∈ ℕ    {multiply_nat_wf}
. |
. |\
. | .....wf .....
. | ⊢  x ∈ ℕ  by Trivial
. |
. .\
. . .....wf .....
. . ⊢  Y((λf,x. if x=₂0→ 1 else x·f(x-1) fi),x-1) ∈ ℕ
. .   by BackThru: Hyp:2
END
```