```
primes_sqr_roots  THM

⊢  ∀a:ℕ. prime(a) ⇒ ¬(∃p:ℕ, q:ℕ+. p•p = a•q•q)  by Auto
|
1. a : ℕ
2. prime(a)
⊢  ¬(∃p:ℕ, q:ℕ+. p•p = a•q•q)
| by BackThru:
|    Thm*  ∀P:(ℕ→Prop).
|          (∀x:ℕ. P(x) ⇒ (∃x':ℕ. x'<x & P(x'))) ⇒ ¬(∃x:ℕ. P(x))
|
|        nat_descent
|
3. p : ℕ
4. ∃q:ℕ+. p•p = a•q•q
⊢  ∃p':ℕ. p'<p & (∃q':ℕ+. p'•p' = a•q'•q')  by Analyze4
|
4. q : ℕ+
5. p•p = a•q•q
⊢  ∃p':ℕ. p'<p & (∃q':ℕ+. p'•p' = a•q'•q')
| by Use:[a]
|    Inst:
|    Thm*  ∀a:ℕ.
|          prime(a)
|          ⇒
|          (∀p:ℕ, q:ℕ+. p•p = a•q•q ⇒ (∃p':ℕ+. p'<p & q•q = a•p'•p'))
|    {primes_sqr_roots_LEMMA1}
|
6. ∀p:ℕ, q:ℕ+. p•p = a•q•q ⇒ (∃p':ℕ+. p'<p & q•q = a•p'•p')
⊢  ∃p':ℕ. p'<p & (∃q':ℕ+. p'•p' = a•q'•q')
| by FwdThru: Hyp:6 on [ Hyp:5 ]
|
7. p' : ℕ+
8. p'<p
9. q•q = a•p'•p'
⊢  ∃p':ℕ. p'<p & (∃q':ℕ+. p'•p' = a•q'•q')
| by FwdThru: Hyp:6 on [ Hyp:9 ]
|
10. q' : ℕ+
11. q'<q
12. p'•p' = a•q'•q'
⊢  ∃p':ℕ. p'<p & (∃q':ℕ+. p'•p' = a•q'•q')
  by Witness: p' THEN Witness: q'
END
```