```
two_sqr_roots  THM

⊢  ¬(∃p:ℕ, q:ℕ+. p·p = 2·q·q)
| by BackThru:
|    Thm*  ∀P:(ℕ→Prop).
|          (∀x:ℕ. P(x) ⇒ (∃x':ℕ. x'<x & P(x'))) ⇒ ¬(∃x:ℕ. P(x))
|
|        nat_descent
|
1. p : ℕ
2. ∃q:ℕ+. p·p = 2·q·q
⊢  ∃p':ℕ. p'<p & (∃q':ℕ+. p'·p' = 2·q'·q')  by Analyze-1
|
2. q : ℕ+
3. p·p = 2·q·q
⊢  ∃p':ℕ. p'<p & (∃q':ℕ+. p'·p' = 2·q'·q')
| by Inst:
|    Thm*  ∀p:ℕ, q:ℕ+. p·p = 2·q·q ⇒ (∃p':ℕ+. p'<p & q·q = 2·p'·p')
|    {two_sqr_roots_LEMMA1}
|
4. ∀p:ℕ, q:ℕ+. p·p = 2·q·q ⇒ (∃p':ℕ+. p'<p & q·q = 2·p'·p')
⊢  ∃p':ℕ. p'<p & (∃q':ℕ+. p'·p' = 2·q'·q')
| by FwdThru: Hyp:4 on [ Hyp:3 ]
|
5. p' : ℕ+
6. p'<p
7. q·q = 2·p'·p'
⊢  ∃p':ℕ. p'<p & (∃q':ℕ+. p'·p' = 2·q'·q')
| by FwdThru: Hyp:4 on [ Hyp:7 ]
|
8. q' : ℕ+
9. q'<q
10. p'·p' = 2·q'·q'
⊢  ∃p':ℕ. p'<p & (∃q':ℕ+. p'·p' = 2·q'·q')
  by Witness: p' THEN Witness: q'
END
```