```
two_sqr_roots_LEMMA1  THM

⊢  ∀p:ℕ, q:ℕ+. p·p = 2·q·q ⇒ (∃p':ℕ+. p'<p & q·q = 2·p'·p')
| by Use:[2]
|    (ApplyDecideLemma
|     Thm*  ∀x:ℤ. Dec(prime(x))    {decidable__prime})
|
1. prime(2)
2. p : ℕ
3. q : ℕ+
4. p·p = 2·q·q
⊢  ∃p':ℕ+. p'<p & q·q = 2·p'·p'
| by FwdThru:
|    Thm*  ∀a:ℤ. prime(a) ⇒ (∀x:ℤ. a | x·x ⟺ a | x)
|    {prime_divides_square}
|    on [ 2 | p·p ]
|
|\
| .....assertion .....
| ⊢  2 | p·p  by Witness: q·q
|
.\
. 5. 2 | p
. ⊢  ∃p':ℕ+. p'<p & q·q = 2·p'·p'  by New:p' Analyze5
. |
. 5. p' : ℤ
. 6. p = 2·p'
. ⊢  ∃p':ℕ+. p'<p & q·q = 2·p'·p'
. | by 2>0 & p>0 & p'>0  By SideProof 1
. |
. 7. 2>0
. 8. p>0
. 9. p'>0
. ⊢  ∃p':ℕ+. p'<p & q·q = 2·p'·p'  by Witness: p'
. |
. ⊢  q·q = 2·p'·p'  by Use:[n:= 2] BackThru: Thm: mul_cancel_in_eq
. |
. ⊢  2·q·q = 2·2·p'·p'
.   by Rewrite by 2·2·p'·p' = 2·p'·2·p' {← hyps Hyp:4 Hyp:6 }
END
```

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two_sqr_roots_LEMMA1 Side Proof 1  THM

1. prime(2)
2. p : ℕ
3. q : ℕ+
4. p·p = 2·q·q
5. p' : ℤ
6. p = 2·p'
⊢  2>0 & p>0 & p'>0
| by ∀a:ℕ, b:ℤ. a·b>0 ⟺ a>0 & b>0
|      By SimilarTo: Thm: pos_mul_arg_bounds
|
7. ∀a:ℕ, b:ℤ. a·b>0 ⟺ a>0 & b>0
⊢  2>0 & p>0 & p'>0
  by 2>0  By Analyze1 THEN 2·q·q>0  By Repeat BackThru: Hyp:7
     THEN
     p>0  By FwdThru: Hyp:7 on [ p·p>0 ]
     THEN
     p'>0  By Sel 2 FwdThru: Hyp:7 on [ 2·p'>0 ]
END
```