```
composite_with_prime_factor  THM

⊢  ∀x:{2...}. ¬prime(x) ⇒ (∃i,j:{2..x⁻}. x = i·j & prime(i))
| by SideProof 1
|
.....assertion .....
⊢  ∀x:ℕ. 2≤x ⇒ ¬prime(x) ⇒ (∃i,j:{2..x⁻}. x = i·j & prime(i))
| by CompNatInd Concl ...ℕ
|
1. x : ℕ
2. ∀x1:ℕ.
|  x1<x ⇒ 2≤x1 ⇒ ¬prime(x1) ⇒ (∃i,j:{2..x1⁻}. x1 = i·j & prime(i))
3. 2≤x
4. ¬prime(x)
⊢  ∃i,j:{2..x⁻}. x = i·j & prime(i)
| by FwdThru:
|    Thm*  ∀x:ℕ. ¬prime(x) ⟺ x<2 ∨ (∃i,j:{2..x⁻}. x = i·j)
|    {nonprime_nat}
|    on [ ¬prime(x) ] ...wℕ
|    THEN
|    Analyze-1 ...ℕ
|
5. ∃i,j:{2..x⁻}. x = i·j
⊢  ∃i,j:{2..x⁻}. x = i·j & prime(i)  by ExistHD Hyp:-1
|
5. i : {2..x⁻}
6. j : {2..x⁻}
7. x = i·j
⊢  ∃i,j:{2..x⁻}. x = i·j & prime(i)  by Decide: prime(i) ...ℕ
|
|\
| 8. prime(i)
| ⊢  ∃i,j:{2..x⁻}. x = i·j & prime(i)  by Witness: i | j ...ℕ
|
.\
. 8. ¬prime(i)
. ⊢  ∃i,j:{2..x⁻}. x = i·j & prime(i)
. | by Inst: Hyp:2 Using:[i] ...ℕ
. |    THEN
. |    CBV-1: ∃n:<type>, m:<type>. <prop>
. |    THEN
. |    ExistHD Hyp:-1
. |
. 9. n : {2..i⁻}
. 10. m : {2..i⁻}
. 11. i = n·m
. 12. prime(n)
. ⊢  ∃i,j:{2..x⁻}. x = i·j & prime(i)  by Witness: n ...wℕ
. |
. ⊢  ∃j:{2..x⁻}. x = n·j & prime(n)  by Rewrite7 by Hyp:11 ...ℕ
. |
. 13. x = n·m·j
. ⊢  ∃j:{2..x⁻}. x = n·j & prime(n)  by Witness: m·j ...ℕ
. |
. .....wf .....
```

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. ⊢  m·j ∈ {2..x-}  by Analyze ...ℕ
. |
. |\
. | ⊢  2≤m·j
. |   by BackThru:
. |      Thm*  ∀x,y:ℕ, z:ℕ+. x≤y ⇒ x≤y·z    {le_mul_rt_by_pos} ...ℕ
. |
. .\
. . ⊢  m·j<x
. . | by Inst: Thm*  ∀i:{2...}, j:ℕ+. j<i·j    {factor_smaller2}
. . |    Using:[n | m·j] ...ℕ
. . |
. . .....wf .....
. . ⊢  m·j ∈ ℕ+
. .   by BackThru: Thm*  ∀x,y:ℕ+. x·y ∈ ℕ+    {mul_nat_plus} ...wℕ
END
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composite_with_prime_factor Side Proof 1  THM

⊢  ∀x:{2...}. ¬prime(x) ⇒ (∃i,j:{2..x-}. x = i·j & prime(i))
| by ∀x:ℕ. 2≤x ⇒ ¬prime(x) ⇒ (∃i,j:{2..x-}. x = i·j & prime(i))
|    Asserted
|
|\
| .....assertion .....
| ⊢  ∀x:ℕ. 2≤x ⇒ ¬prime(x) ⇒ (∃i,j:{2..x-}. x = i·j & prime(i))
|   by <Unproved PREMISE>
|
.\
. 1. ∀x:ℕ. 2≤x ⇒ ¬prime(x) ⇒ (∃i,j:{2..x-}. x = i·j & prime(i))
. ⊢  ∀x:{2...}. ¬prime(x) ⇒ (∃i,j:{2..x-}. x = i·j & prime(i))
. | by Autoℕ
. |
. 2. x : {2...}
. 3. ¬prime(x)
. ⊢  ∃i,j:{2..x-}. x = i·j & prime(i)  by BackThru: Hyp:1 ...ℕ
END
```