```
divides_def_on_nat  THM

⊢  ∀a,b:ℕ. a | b ⟺ (∃c:ℕ. b = a•c)
| by Def of a | b THEN UnivCD ...wℕ
|
1. a : ℕ
2. b : ℕ
⊢  (∃c:ℤ. b = a•c) ⟺ (∃c:ℕ. b = a•c)  by Autoℕ
|
|\
| 3. ∃c:ℤ. b = a•c
| ⊢  ∃c:ℕ. b = a•c  by Analyze-1
| |
| 3. c : ℤ
| 4. b = a•c
| ⊢  ∃c:ℕ. b = a•c  by Decide: c<0 ...ℕ
| |
| |\
| | 5. c<0
| | ⊢  ∃c:ℕ. b = a•c  by b = 0  Asserted
| | |
| | |\
| | | .....assertion .....
| | | ⊢  b = 0 ∈ ℕ
| | |   by Inst: Thm*  ∀a,b:ℕ. 0≤a•b    {mul_bounds_1a}
| | |      Using:[a | -c] ...ℕ
| | |
| | .\
| | . 6. b = 0 ∈ ℕ
| | . ⊢  ∃c:ℕ. b = a•c  by Witness: 0 ...ℕ
| |
| .\
| . 5. ¬c<0
| . ⊢  ∃c:ℕ. b = a•c  by Witness: c ...ℕ
|
.\
. 3. ∃c:ℕ. b = a•c
. ⊢  ∃c:ℤ. b = a•c  by SimilarTo: Hyp:-1 ...wℕ
END
```