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prime_among_factors  THM

⊢  ∀a,b:ℕ. prime(a·b) ⇒ a·b = a ∨ a·b = b
| by Autoℕ
|    THEN
|    (Rewrite'2 by
|     Thm*  ∀x:ℕ. prime(x) ⟺ 2≤x & ¬(∃i:{2..x⁻}. i | x)
|     {prime_nat} ...ℕ ;
|     Id)
|
1. a : ℕ
2. b : ℕ
3. prime(a·b)
⊢  a·b = a ∨ a·b = b
| by Guarding (<prop> & <prop>)
|    FwdThru:
|    Thm*  ∀x:ℕ. prime(x) ⟺ 2≤x & ¬(∃i:{2..x⁻}. i | x)    {prime_nat}
|    on [ prime(a·b) ] ...ℕ
|
4. 2≤a·b
5. ¬(∃i:{2..(a·b)⁻}. i | a·b)
⊢  a·b = a ∨ a·b = b
| by Enough that
|    ¬(¬a·b = a ∈ ℤ & ¬a·b = b ∈ ℤ)
|      By (FwdThru:
|          Thm*  Dec(A) ⇒ (¬(A & B) ⟺ ¬A ∨ ¬B)    {not_over_and}
|          on [ Hyp:-1 ] ...ℕ)
|
.....assertion .....
⊢  ¬(¬a·b = a & ¬a·b = b)  by SimilarTo Hyp:5 ...ℕ
|
5. ¬a·b = a
6. ¬a·b = b
⊢  ∃i:{2..(a·b)⁻}. i | a·b
| by Guarding (<prop> & <prop>)
|    FwdThru:
|    Thm*  ∀a,b:ℕ. 0<a·b ⟺ 0<a & 0<b    {pos_mul_arg_boundsNat}
|    on [ 0<a·b ] ...ℕ
|    THEN
|    Inst: Thm*  ∀i,j:ℕ⁺. i≤i·j & j≤i·j    {factors_bound}
|    Using:[a | b] ...ℕ
|
7. 0<a
8. 0<b
9. a≤a·b
10. b≤a·b
⊢  ∃i:{2..(a·b)⁻}. i | a·b  by ¬a = 1  Asserted ...ℕ
|
|\
| .....assertion .....
| ⊢  ¬a = 1  by SimilarTo: Hyp:6 ...wℕ
| |
| 11. a = 1
| ⊢  a·b = b  by Rewrite by a = 1 ...ℕ
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. 11. ¬a = 1
. ⊢  ∃i:{2..(a·b)⁻}. i | a·b  by Witness: a | b ...ℕ
END
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