```
prime_nat  THM

⊢  ∀x:ℕ. prime(x) ⟺ 2≤x & ¬(∃i:{2..x⁻}. i | x)  by Auto
|
|\
| 1. x : ℕ
| 2. prime(x)
| ⊢  2≤x
| | by Use:[x]
| |    Inst:
| |    Thm*  ∀p:ℤ.
| |          prime(p)
| |          ⇒
| |          ¬p = 0 & ¬(p ∼ 1) & (∀a:ℤ. a | p ⇒ (a ∼ 1) ∨ (a ∼ p))
| |    {prime_elim}
| |
| 3. ¬x = 0
| 4. ¬(x ∼ 1)
| 5. ∀a:ℤ. a | x ⇒ (a ∼ 1) ∨ (a ∼ x)
| ⊢  2≤x
|   by ¬x = 1  Asserted THEN SimilarTo: ¬(x ∼ 1)
|      THEN
|      BackThru: Thm*  ∀a,b:ℕ. (a ∼ b) ⟺ a = b    {assoced_nelim}
|
|\
| 1. x : ℕ
| 2. prime(x)
| ⊢  ¬(∃i:{2..x⁻}. i | x)  by SimilarTo: ¬(∃i:{2..x⁻}. i = 1 ∨ i = x)
| |
| |\
| | .....assertion .....
| | ⊢  ¬(∃i:{2..x⁻}. i = 1 ∨ i = x)  by Analyze THEN Analyze3
| |
| .\
| . 3. i : {2..x⁻}
| . 4. i | x
| . ⊢  i = 1 ∨ i = x
| .   by SimilarTo (i ∼ 1) ∨ (i ∼ x)
| .      THEN
| .      Backchain
| .      [Thm*  ∀a,b:ℕ. (a ∼ b) ⟺ a = b    {assoced_nelim}
| .      ;Thm*  ∀p:ℤ.
| .             prime(p)
| .             ⇒
| .             ¬p = 0 & ¬(p ∼ 1) & (∀a:ℤ. a | p ⇒ (a ∼ 1) ∨ (a ∼ p))
| .       {prime_elim}]
|
.\
. 1. x : ℕ
. 2. 2≤x
. 3. ¬(∃i:{2..x⁻}. i | x)
. ⊢  prime(x)
. | by BackThru:
. |    Thm*  ∀a:ℤ. atomic(a) ⇒ prime(a)    {atomic_imp_prime}
. |
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```
. ⊢  atomic(x)
. | by BackThru:
. |    Thm*  ∀a:ℤ.
. |          atomic(a)
. |          ⟺
. |          ¬(a ∼ 1) & (∀b:ℤ. b | a ⇒ (b ∼ 1) ∨ (b ∼ a))
. |    {atomic_char}
. |
. |\
. | ⊢  ¬(x ∼ 1)
. |   by SimilarTo: ¬x = 1
. |      THEN
. |      BackThru: Thm*  ∀a,b:ℕ. (a ∼ b) ⟺ a = b    {assoced_nelim}
. |
. .\
. . 4. b : ℤ
. . 5. b | x
. . ⊢  (b ∼ 1) ∨ (b ∼ x)  by SideProof 1
. . |
. . .....assertion .....
. . 5. |b| | x
. . 6. |b| ∼ b
. . ⊢  (|b| ∼ 1) ∨ (|b| ∼ x)
. . | by Rewrite by Thm*  ∀a,b:ℕ. (a ∼ b) ⟺ a = b    {assoced_nelim}
. . |
. . ⊢  |b| = 1 ∨ |b| = x
. . | by FwdThru:
. . |    Thm*  ∀a:ℕ, b:ℕ+. a | b ⇒ a≤b    {divisors_bound}
. . |    on [ Hyp:5 ]
. . |
. . 7. |b|≤x
. . ⊢  |b| = 1 ∨ |b| = x  by ¬(2≤|b| & |b|<x)  Asserted
. . |
. . |\
. . | .....assertion .....
. . | ⊢  ¬(2≤|b| & |b|<x)  by SimilarTo Hyp:3
. . | |
. . | 8. 2≤|b| & |b|<x
. . | ⊢  ∃i:{2..x⁻}. i | x  by Witness: |b|
. . |
. . .\
. . . 8. ¬(2≤|b| & |b|<x)
. . . ⊢  |b| = 1 ∨ |b| = x  by ¬|b| = 0  Asserted
. . . |
. . . |\
. . . | .....assertion .....
. . . | ⊢  ¬|b| = 0  by Analyze
. . . | |
. . . | 9. |b| = 0
. . . | ⊢  False
. . . |   by FwdThru:
. . . |      Thm*  ∀a:ℤ. 0 | a ⇒ a = 0    {zero_divs_only_zero}
. . . |      on [ 0 | x ]
. . . |      THENA
. . . |      Subst←: |b| = 0
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. . . |
. . . .\
. . . . 9. ¬|b| = 0
. . . . ⊢  |b| = 1 ∨ |b| = x
. . . .   by FwdThru:
. . . .      Thm*  Dec(A) ⇒ (¬(A & B) ⟺ ¬A ∨ ¬B)    {not_over_and}
. . . .      on [ ¬(2≤|b| & |b|<x) ]
END
prime_nat Side Proof 1  THM

1. x : ℕ
2. 2≤x
3. ¬(∃i:{2..x⁻}. i | x)
4. b : ℤ
5. b | x
⊢  (b ∼ 1) ∨ (b ∼ x)
| by |b| | x  Asserted
|    THENA
|    (Subst: x = |x|
|     THENA
|     BackThru: Thm*  ∀i:ℕ. |i| = i    {absval_pos}
|     THEN
|     Sel 2
|     BackThru:
|     Thm*  ∀a,b:ℤ. |a| | |b| ⟺ a | b    {divides_of_absvals})
|
5. |b| | x
⊢  (b ∼ 1) ∨ (b ∼ x)
| by (Use:[b] Inst: Thm*  ∀a:ℤ. |a| ∼ a    {absval_assoced}
|     THEN
|     SimilarTo (|b| ∼ 1) ∨ (|b| ∼ x))
|    THEN
|    ChainRelation:
|    Thm*  EquivRel x,y:ℤ. x ∼ y    {assoced_equiv_rel}
|
.....assertion .....
6. |b| ∼ b
⊢  (|b| ∼ 1) ∨ (|b| ∼ x)  by <Unproved PREMISE>
END
```