```
factorial_wf  THM

⊢  ∀n:ℕ. (n)!  ∈ ℕ  by Analyze 0
|
1. n : ℕ
⊢  (n)!  ∈ ℕ  by NatInd 1
|
|\
| .....basecase .....
| (no hyps)
| ⊢  (0)!  ∈ ℕ  by Auto
| |
| |\
| | ⊢  (0)!  ∈ ℤ  by RecCaseSplit ‘factorial‘
| | |
| | |\
| | | .....truecase .....
| | | 1. 0 = 0
| | | ⊢  1 ∈ ℤ  by Auto
| | |
| | .\
| | . 1. ¬0 = 0
| | . ⊢  0·(-1)!  ∈ ℤ  by Auto
| |
| .\
| . .....set predicate .....
| . ⊢  0≤(0)!   by RecCaseSplit ‘factorial‘
|
.\
. .....upcase .....
. 1. n : ℤ
. 2. 0<n
. 3. (n-1)!  ∈ ℕ
. ⊢   by RecCaseSplit ‘factorial‘
. |
. |\
. | 4. ¬n = 0
. | ⊢  (n-1)!  ∈ ℤ
. |   by Inst Thm* ∀n:ℕ. n ∈ ℤ    {nat_is_int} [(n-1)! ]
. |
. .\
. . .....set predicate .....
. . 4. ¬n = 0
. . ⊢  0≤n·(n-1)!
. . | by Inst Thm* ∀a,b:ℤ, n:ℕ. a≤b ⇒ n·a≤n·b    {mul_preserves_le}
. . |    [0;n;(n-1)! ]
. . |
. . |\
. . | .....antecedent .....
. . | ⊢  0≤n  by SupInf
. . |
. . .\
. . . 5. (n-1)! ·0≤(n-1)! ·n
. . . ⊢   by Inst Thm* ∀a,b:ℤ. a·b = b·a    {mul_com} [n;(n-1)! ]
. . . |
```

```
. . . |\
. . . | .....wf .....
. . . | ⊢  (n-1)!  ∈ ℤ
. . . |   by Inst Thm* ∀n:ℕ. n ∈ ℤ    {nat_is_int} [(n-1)! ]
. . . |
. . . .\
. . . . 6. n·(n-1)!  = (n-1)! ·n
. . . . ⊢   by Auto
END
```