pump_thm_cor THM

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⊢  ∀n:ℕ+, Alph:Type, S:ActionSet(Alph), s,f:S.car.
|  #(S.car)=n  ⇒
|  (∃l:Alph*. (S:l←s) = f) ⇒ (∃l:Alph*. ||l||≤n & (S:l←s) = f)
| by UnivCD
|
1. n : ℕ+
2. Alph : Type
3. S : ActionSet(Alph)
4. s : S.car
5. f : S.car
6. #(S.car)=n
7. ∃l:Alph*. (S:l←s) = f
⊢  ∃l:Alph*. ||l||≤n & (S:l←s) = f
| by Assert
|    (∀k:ℕ.
|     (∃l:Alph*. ||l||<k & (S:l←s) = f) ⇒
|     (∃l:Alph*. ||l||≤n & (S:l←s) = f))
|
|\
| .....assertion .....
| ⊢  ∀k:ℕ.
| |  (∃l:Alph*. ||l||<k & (S:l←s) = f) ⇒
| |  (∃l:Alph*. ||l||≤n & (S:l←s) = f)  by Analyze 0
| |
| 8. k : ℕ
| ⊢  (∃l:Alph*. ||l||<k & (S:l←s) = f) ⇒
| |  (∃l:Alph*. ||l||≤n & (S:l←s) = f)  by NatInd 8
| |
| |\
| | .....basecase .....
| | 7. ∃l:Alph*. (S:l←s) = f
| | ⊢  (∃l:Alph*. ||l||<0 & (S:l←s) = f) ⇒
| | |  (∃l:Alph*. ||l||≤n & (S:l←s) = f)  by Analyze 0
| | |
| | 8. ∃l:Alph*. ||l||<0 & (S:l←s) = f
| | ⊢  ∃l:Alph*. ||l||≤n & (S:l←s) = f  by Analyze 8
| |
| .\
| . .....upcase .....
| . 8. k : ℤ
| . 9. 0<k
| . 10. (∃l:Alph*. ||l||<k-1 & (S:l←s) = f) ⇒
| . |   (∃l:Alph*. ||l||≤n & (S:l←s) = f)
| . ⊢   by Analyze 0
| . |
| . 11. ∃l:Alph*. ||l||<k & (S:l←s) = f
| . ⊢  ∃l:Alph*. ||l||≤n & (S:l←s) = f  by Analyze 11
| . |
| . 11. l : Alph*
| . 12. ||l||<k & (S:l←s) = f
| . ⊢   by Analyze 12
| . |
| . 12. ||l||<k
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| . 13. (S:l←s) = f
| . ⊢
| . | by Inst
| . |    Thm* ∀S:ActionSet(Alph), N:ℕ+, s:S.car.
| . |         #(S.car)=N  ⇒
| . |         (∀A:Alph*.
| . |          N<||A|| ⇒
| . |          (∃a,b,c:Alph*.
| . |           0<||b|| & A = ((a @ b) @ c)
| . |           & (∀k:ℕ. (S:(a @ (b↑k)) @ c←s) = (S:A←s))))
| . |    {pump_theorem}
| . |    [Alph;S;n;s]
| . |
| . 14. ∀A:Alph*.
| . |   n<||A|| ⇒
| . |   (∃a,b,c:Alph*.
| . |    0<||b|| & A = ((a @ b) @ c)
| . |    & (∀k:ℕ. (S:(a @ (b↑k)) @ c←s) = (S:A←s)))
| . ⊢   by Decide (||l||≤n)
| . |
| . |\
| . | 15. ||l||≤n
| . | ⊢   by Witness l
| . |
| . .\
| . . 15. ¬||l||≤n
| . . ⊢   by Witness14 l
| . . |
| . . 14. ¬||l||≤n
| . . 15. n<||l|| ⇒
| . . |   (∃a,b,c:Alph*.
| . . |    0<||b|| & l = ((a @ b) @ c)
| . . |    & (∀k:ℕ. (S:(a @ (b↑k)) @ c←s) = (S:l←s)))
| . . ⊢   by Analyze 15
| . . |
| . . 15. ∃a,b,c:Alph*.
| . . |   0<||b|| & l = ((a @ b) @ c)
| . . |   & (∀k:ℕ. (S:(a @ (b↑k)) @ c←s) = (S:l←s))
| . . ⊢
| . . | by Analyze 15 THEN Analyze 16 THEN Analyze 17 THEN Analyze 18
| . . |
| . . 15. a : Alph*
| . . 16. b : Alph*
| . . 17. c : Alph*
| . . 18. 0<||b|| & l = ((a @ b) @ c)
| . . 19. ∀k:ℕ. (S:(a @ (b↑k)) @ c←s) = (S:l←s)
| . . ⊢   by Witness19 0

| . . |
| . . 19. (S:(a @ (b↑0)) @ c←s) = (S:l←s)
| . . ⊢
| . . | by RWH
| . . |    (LemmaC
| . . |     Thm* ∀L:Alph*, n:ℕ.
| . . |          (L↑n) = if n=₂0→ nil else L @ (L↑n-1) fi
| . . |     {lpower_alt})
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| . . |    19
| . . |
| . . 19. (S:(a @ if 0=_{2}0→ nil else b @ (b↑0-1) fi) @ c←s) = (S:l←s)
| . . ⊢   by Reduce 19
| . . |
| . . 19. (S:(a @ nil) @ c←s) = (S:l←s)
| . . ⊢
| . . | by RWH
| . . |    (LemmaC Thm* ∀as:T*. (as @ nil) = as    {append_back_nil})
| . . |    19
| . . |
| . . 19. (S:a @ c←s) = (S:l←s)
| . . ⊢   by Analyze 10
| . . |
| . . |\
| . . | .....antecedent .....
| . . | 10. l : Alph*
| . . | 11. ||l||<k
| . . | 12. (S:l←s) = f
| . . | 13. ¬||l||≤n
| . . | 14. a : Alph*
| . . | 15. b : Alph*
| . . | 16. c : Alph*
| . . | 17. 0<||b|| & l = ((a @ b) @ c)
| . . | 18. (S:a @ c←s) = (S:l←s)
| . . | ⊢  ∃l:Alph*. ||l||<k-1 & (S:l←s) = f  by Analyze 17
| . . | |
| . . | 17. 0<||b||
| . . | 18. l = ((a @ b) @ c)
| . . | 19. (S:a @ c←s) = (S:l←s)
| . . | ⊢   by Witness a @ c
| . . | |
| . . | ⊢  ||a @ c||<k-1 & (S:a @ c←s) = f  by Auto
| . . | |
| . . | ⊢  ||a @ c||<k-1
| . . | | by RWH
| . . | |    (LemmaC
| . . | |     Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
| . . | |     {length_append})
| . . | |    0
| . . | |
| . . | ⊢  ||a||+||c||<k-1  by ApFunToHypEquands ‘x’ ||x|| ℤ 18
| . . | |
| . . | 20. ||l|| = ||(a @ b) @ c||
| . . | ⊢
| . . |   by RWH
| . . |      (LemmaC
| . . |       Thm* ∀as,bs:T*. ||as @ bs|| = ||as||+||bs||
| . . |       {length_append})
| . . |      20
| . . |
| . . .\
| . . . 10. l : Alph*
| . . . 11. ||l||<k
| . . . 12. (S:l←s) = f
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| . . . 13. ¬||l||≤n
| . . . 14. a : Alph*
| . . . 15. b : Alph*
| . . . 16. c : Alph*
| . . . 17. 0<||b|| & l = ((a @ b) @ c)
| . . . 18. (S:a @ c←s) = (S:l←s)
| . . . 19. ∃l:Alph*. ||l||≤n & (S:l←s) = f
| . . . ⊢   by Auto
|
.\
. 8. ∀k:ℕ.
. |  (∃l:Alph*. ||l||<k & (S:l←s) = f) ⇒
. |  (∃l:Alph*. ||l||≤n & (S:l←s) = f)
. ⊢   by Analyze 7
. |
. 7. l : Alph*
. 8. (S:l←s) = f
. 9. ∀k:ℕ.
. |  (∃l:Alph*. ||l||<k & (S:l←s) = f) ⇒
. |  (∃l:Alph*. ||l||≤n & (S:l←s) = f)
. ⊢   by Witness9 ||l||+1
. |
. 9. (∃l@0:Alph*. ||l@0||<||l||+1 & (S:l@0←s) = f) ⇒
. |  (∃l:Alph*. ||l||≤n & (S:l←s) = f)
. ⊢   by Analyze 9
. |
. |\
. | .....antecedent .....
. | 8. (S:l←s) = f
. | ⊢  ∃l@0:Alph*. ||l@0||<||l||+1 & (S:l@0←s) = f  by Witness l
. | |
. | ⊢  ||l||<||l||+1 & (S:l←s) = f  by Auto
. |
. .\
. . 9. ∃l:Alph*. ||l||≤n & (S:l←s) = f
. . ⊢   by Auto
END
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