```
fib_coprime_sfa2  THM

⊢  ∀n:ℕ. CoPrime(fib(n),fib(n+1))
| by SimilarTo ∀n,k:ℕ. k = n+1 ⇒ CoPrime(fib(n),fib(k))
|    {We rephrase the theorem to simplify the arithhmetic expressions
|     that will arise in the proof. }
|
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| .....assertion .....
| ⊢  ∀n,k:ℕ. k = n+1 ∈ ℕ ⇒ CoPrime(fib(n),fib(k))
| | by Rewrite by
| |    Thm* ∀a,b:ℤ. CoPrime(a,b) ⟺ (∃x,y:ℤ. a·x+b·y = 1)
| |    {coprime_bezout_id}
| |    THEN
| |    CompNatInd Concl
| |
| 1. n : ℕ
| 2. ∀n1:ℕ.
| |  n1<n ⇒ (∀k:ℕ. k = n1+1 ∈ ℕ ⇒ (∃x,y:ℤ. fib(n1)·x+fib(k)·y = 1))
| 3. k : ℕ
| 4. k = n+1 ∈ ℕ
| ⊢  ∃x,y:ℤ. fib(n)·x+fib(k)·y = 1
| | by Compute
| |      ⌈fib(k)⌉ *⇒ if k=₂0 ∨₂ k=₂1→ 1 else fib(k-1)+fib(k-2) fi
| |    THEN
| |    SplitOnConclITE
| |    THEN
| |    CBV: ∃a,b:ℤ. <prop>
| |
| |\
| | 5. k = 0 ∨ k = 1
| | ⊢  ∃a,b:ℤ. fib(n)·a+1·b = 1  by Witness: 0 | 1
| |
| .\
| . 5. ¬k = 0
| . 6. ¬k = 1
| . ⊢  ∃a,b:ℤ. fib(n)·a+(fib(k-1)+fib(k-2))·b = 1
| . | by Rewrite by n = k-1 ∈ ℕ
| . |
| . ⊢  ∃a,b:ℤ. fib(k-1)·a+(fib(k-1)+fib(k-2))·b = 1
| . | by SimilarTo: ∃a,b:ℤ. fib(k-2)·b+fib(k-1)·(a+b) = 1
| . |
| . .....assertion .....
| . ⊢  ∃a,b:ℤ. fib(k-2)·b+fib(k-1)·(a+b) = 1
| . | by Use:[k-2 | k-1] Inst: Hyp:2
| . |
| . 7. ∃x,y:ℤ. fib(k-2)·x+fib(k-1)·y = 1
| . ⊢  ∃a,b:ℤ. fib(k-2)·b+fib(k-1)·(a+b) = 1
| .   by ExistHD Hyp:-1 THEN Witness: y-x | x
|
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. 1. n : ℕ
. 2. ∀k:ℕ. k = n+1 ∈ ℕ ⇒ CoPrime(fib(n),fib(k))
. ⊢  CoPrime(fib(n),fib(n+1))  by BackThru: Hyp:-1
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END