```
fib_coprime_sfa3  THM

⊢  ∀n:ℕ. CoPrime(fib(n),fib(n+1))
| by Rewrite by
|    Thm* ∀a,b:ℤ. CoPrime(a,b) ⟺ (∃x,y:ℤ. a·x+b·y = 1)
|    {coprime_bezout_id}
|    THEN
|    CompNatInd Concl
|
1. n : ℕ
2. ∀k:ℕ. k<n ⇒ (∃x,y:ℤ. fib(k)·x+fib(k+1)·y = 1)
⊢  ∃a,b:ℤ. fib(n)·a+fib(n+1)·b = 1
| by Compute
|      ⌈fib(n+1)⌉
|      *⇒
|      if n+1=₂0 ∨₂ n+1=₂1→ 1 else fib(n+1-1)+fib(n+1-2) fi
|    THEN
|    SplitOnConclITE
|
|\
| ⊢  ∃a,b:ℤ. fib(n)·a+1·b = 1  by Witness: 0 | 1
|
.\
. 3. ¬n+1 = 1
. ⊢  ∃a,b:ℤ. fib(n)·a+(fib(n)+fib(n-1))·b = 1
. | by SimilarTo: ∃a,b:ℤ. fib(n-1)·b+fib(n)·(a+b) = 1
. |
. ⊢  ∃a,b:ℤ. fib(n-1)·b+fib(n)·(a+b) = 1
. | by Inst: Hyp:2 Using:[n-1] THEN Subst-1: n-1+1 = n ∈ ℕ
. |
. 4. ∃x,y:ℤ. fib(n-1)·x+fib(n)·y = 1
. ⊢  ∃a,b:ℤ. fib(n-1)·b+fib(n)·(a+b) = 1
. | by ExistHD Hyp:-1 THEN Witness: y-x | x
. |
. 4. x : ℤ
. 5. y : ℤ
. 6. fib(n-1)·x+fib(n)·y = 1
. ⊢  fib(n-1)·x+fib(n)·(y-x+x) = 1  by Auto
END
```